# Investigation on Arrangement of Sound Source Elements to Improve Results of Reflection Point Search by Rectangular Sound Source <br> 矩形音源による反射点探索結果を改善するための音源要素配置に関する検討 

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## 1．Introduction

In ultrasonic measuring devices and imaging equipment，rectangular transducers are widely used as elements of the sound sources．Rectangular transducers have four vertices and four sides，and a spatial impulse response of a rectangular sound source changes complicatedly depending on the order in which edge waves from these vertices and sides and direct waves from the sound source surface arrive at the observation point ${ }^{1)}$ ．In addition， the waveform acquired by a rectangular sound source changes depending on the position of the observation point，subject to the spatial impulse response．A method to apply this complicated change to the reflection point search using a single rectangular sound source ${ }^{2,3)}$ or a rectangular array sound source with a small number of elements ${ }^{4-10)}$ has been proposed．

In the conventional methods using these sound sources，when the reflection point is around the perpendicular from the center of the sound source to the observation space，which the direct wave from the surface of the sound source reaches， the arriving reflection waveform is strongly influ－ enced by the direct wave．This is considered to be one of the reasons why good search results of re－ flection points cannot be obtained in that region．

In this study，as a method of reducing the in－ fluence of direct wave，a sound source with ele－ ments arranged in a rectangular ring is introduced． Numerical calculations of cross－correlation coeffi－ cients are performed to investigate the improvement of the reflection point search results．

## 2．Method of Reflection Point Search

The configuration of a sound source with elements arranged in a rectangular ring and a re－ flection point $P$ is shown in Fig．1．The sound source is assigned to a plane perpendicular to the $z$－axis，and the center of the whole sound source is located at the origin of the coordinates．Each sound source element is arranged offset from the origin， and forms a rectangular ring as a whole．The center of each element is expressed as（ $x_{o f f}, y_{o f f}$ ）．The po－


Fig． 1 Configuration of a sound source with elements arranged in a rectangular ring and a reflection point $P$ ．
sition of the reflection point is indicated by $P(r)$ ．In the calculation result showing in the following sec－ tion，$r$ is expressed using the distance from the origin of the coordinates $(|\boldsymbol{r}|)$ ，the azimuth angle， and the elevation angle．

When each sound source element is driven with uniform velocity $v(t)$ ，and when the wave radi－ ated from the sound source is reflected at $P$ ，the output $e_{n}(\boldsymbol{r}, t)$ of the reflected wave received by the element $n$ is expressed as ${ }^{11)}$

$$
\begin{align*}
& e_{n}(\boldsymbol{r}, t) \\
& \quad=-\frac{k \rho A}{2 c} v(t) * \frac{\partial}{\partial t} h\left(\boldsymbol{r}_{n}, t\right) * \frac{\partial}{\partial t} h\left(\boldsymbol{r}_{n}, t\right), \tag{1}
\end{align*}
$$

where $k$ is the proportionality constant，$\rho$ is the den－ sity of the propagation medium of the sound wave， $A$ is the area of the region in which the reflection point contributes to the reflection，$c$ is the velocity of sound，$h(\cdot)$ is the spatial impulse response of the sound source，and $*$ denotes the convolution inte－ gral．Additionally， $\boldsymbol{r}_{n}$ represents the position of the reflection points from the center of each sound source element，respectively．

The rise time of the reflected wave is measur－ able．Therefore，the value of $|\boldsymbol{r}|$ can be determined in the range expressed as

$$
\begin{equation*}
\frac{c T}{2}<|\boldsymbol{r}|<\frac{c T}{2}+\sqrt{\left(\left|x_{o f f}\right|+\frac{a}{2}\right)^{2}+\left(\left|y_{\text {off }}\right|+\frac{b}{2}\right)^{2}}, \tag{2}
\end{equation*}
$$

where $T$ is the rise time of the reflected wave，and $c$


Fig. 2 Calculation results of cross-correlation coefficients at three reflection points: (I) using sound source with elements arranged in a rectangular ring; (II) using single rectangular sound source.
is velocity of sound. When the value of $\boldsymbol{r}$ is set at an appropriate interval in the range of $|\boldsymbol{r}|$, the spatial impulse response $h\left(\boldsymbol{r}_{n}, t\right)$ corresponding to each $\boldsymbol{r}_{n}$ can be obtained. Since $v(t)$ is known, the output waveform $e_{n}(\boldsymbol{r}, t)$ in eq. (1) at each $\boldsymbol{r}$ can be calculated. By deducing the cross-correlation coefficient between the waveform obtained by the calculation and the original (acquired) reflected wave in the sequential order, it becomes possible to estimate the position of the reflection point $P$.

## 3. Numerical Calculations

The results of numerical calculations by the sound source with elements arranged in a rectangular ring are shown in Fig. 2(I). The results are obtained by calculating convolution integral in eq. (1) and the cross-correlation coefficient at time zero with the calculation result for the points around the reflection points sequentially. The dimensions of the sound source used in the calculation are $a=$ 6.450 mm , and $b=10.050 \mathrm{~mm}$. The dimension of each element is $a / 2 \times b$ (for elements 1 and 3 ) and $2 a \times b / 2$ (for elements 2 and 4). The centers of elements are $\left(x_{o f f 1}, y_{o f f 1}\right)=(3 a / 4,0),\left(x_{o f 2}, y_{o f f 2}\right)=(0$, $3 b / 4),\left(x_{o f f}, y_{o f \beta}\right)=(-3 a / 4,0)$, and $\left(x_{o f f}, y_{o f f}\right)=(0$, $-3 b / 4)$, respectively. The convolution integral and the correlation coefficient are calculated in each receiving element separately, and the average is taken. For the comparison, the calculation results by a single rectangular sound source which has the dimension of $2 a \times 2 b^{2)}$ are also shown in Fig. 2(II).

In Fig. 2(I), the obvious stripe patterns due to
the fluctuation of the correlation coefficient that appear in Fig. 2 (II) are smoothed, and it is considered that the influences of the direct wave observed in the search results with the conventional method using a single sound source element are suppressed.

## 4. Summary

In the reflection point search by rectangular sound sources, the sound source with elements arranged in a rectangular ring was introduced. From the calculation results of the correlation coefficients, it was shown that the influence of the direct wave which deteriorated the search results by the conventional method is suppressed, and that the proposed method is effective for improving search results.

In order to obtain better search results, it is necessary to further consider the number and arrangement of elements.

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