Source-State-Controlled Equivalent Circuit for Electromechanical Transducer

電源状態制御を加えた電気機械変換等価回路

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1. Introduction

For an electromechanical transducer, Mason's equivalent circuit can successfully represent the behavior of the resonance observed from the mechanical (M-) source of the transducer, which is influenced by the impedance connected to the electrical (E-) source or load. However, when the behavior is observed from the E-source, especially when the source is not regarded as 'ideal voltage source', some inconveniences occur, and in this study, a revised equivalent circuit is newly introduced.

2. The conventional circuit structure

Figure 1 shows an essential framework of the equiva-



Fig. 1: Conventional equivalent circuit, when the energy is changed (a) from electrical to mechanical state, and (b) from mechanical to electrical state. *C* is given by $\lim_{\omega \to 0} Z_{\text{LC}} = 1/(j\omega C)$.

lent circuit based on Mason's one. In Fig. 1(a), $S_{\rm E}$ and $Z_{\rm M}$ indicate an electrical (E-) voltage source and amechanical (M-) load, respectively, while in Fig. 1(b), $S_{\rm M}$ and $Z_{\rm E}$ indicate an equivalent M-source and an E-load, respectively. $Z_{\rm E}$ also behaves as an internal impedance in the E-source. $Z_{\rm LC}$ is a pure or intrinsic M-resonance component. The pure or intrinsic elastic equivalent capacitance C is derived from the low frequency limit of $Z_{\rm LC}$, and C_0 is a dielectric capacitance; the ratio C/C_0 determines the electromechanical coupling coefficient k^2 . The difference between the longitudinal (L-) effect (e.g. piezoelectric 33-coupling) and transverse (T-) effect (e.g. piezoelectric 31-coupling) appears in $-C_0/\alpha$, as

$$\alpha = \begin{cases} 1 & \text{(existing and valid) for L-effect,} \\ 0 & \text{(shorted out and void) for T-effect.} \end{cases}$$
(1)

The state of the E-source or load controls the acoustic velocity; in other words, the elastic stiffness c in the transducer ($c^{\rm D}$ for $Z_{\rm E} \to \infty$, and $c^{\rm E}$ for $Z_{\rm E} \to 0$), related with the circuit parameters C and C_0 as

- (i) $c^{\rm D} \sim 1/C$ for L-effect $(Z_{\rm E} \to \infty)$;
- (ii) $c^{\rm E} \sim 1/C 1/C_0$ for L-effect ($Z_{\rm E} \rightarrow 0$);
- (iii) $c^{\rm D} \sim 1/C + 1/C_0$ for T-effect $(Z_{\rm E} \to \infty)$;
- (iv) $c^{\rm E} \sim 1/C$ for T-effect ($Z_{\rm E} \rightarrow 0$).

However, when the admittance is observed from the E-source, the usage of the circuit is practically limited to the case of $Z_E \rightarrow 0$. In other words, the admittance does not reflect the correct properties of the measured object unless $Z_E \rightarrow 0$.

Let us consider an example shown in Fig. 2, in which the condition of $Z_E \rightarrow 0$ is not satisfied, due to the existence of vacuum layers. In this configuration, the transducer is driven electrically with the elastic



Fig. 2: An example in which the transducer is driven with electrically opened state via vacuum layers.



Fig. 3: Arrows indicating the way of current flow when the transducer is driven electrically under the condition of (a) elastic stiffness given by $c^{\rm D}$ ($Z_{\rm E} \rightarrow \infty$) and (b) elastic stiffness given by $c^{\rm E}$ ($Z_{\rm E} \rightarrow 0$).

stiffness of $c^{\rm D}$, with the way of current flow shown in Fig. 3(a), while observing the admittance from the E-source gives the elastic stiffness $c^{\rm E}$, as in Fig. 3(b).

3. Revised equivalent circuit model

For the purpose of removing the physical and practical inconveniences mentioned above, revised circuit models shown in Figs. 4(a) and 4(b) are proposed as replacements for Figs. 1(a) and 1(b), respectively. The function of source is considered in two steps: The energy from the first source in the E-part is transferred into the secondary source in the M-part in Fig. 4(a), and inversely in Fig. 4(b). In Fig. 4(a), the voltage at C_0 in the E-part, V_1 , is transferred to the secondary voltage source in the M-part with "internal capaci-





Fig. 4: Revised equivalent circuit for (a) transduction from electrical to mechanical energy and (b) transduction from mechanical to electrical energy.

tance" \tilde{C}_0 as

$$\tilde{C}_0 = C_0/(1-\beta), \quad \beta = 1/(1+j\omega C_0 Z_{\rm E}).$$
 (2)

In Fig. 4(b), the current in the M-part, I_1 , is transferred to the secondary <u>current source</u> in the E-part with internal capacitance C_0 . This secondary current source provides the E-load with current $I_2 = \beta I_1$.

Both in Fig. 4(a) and 4(b), due to the existence of \tilde{C}_0 , the inverse of equivalent capacitance around the M-part circuit, $1/C_{\rm mech}$, is given by

$$\frac{1}{C_{\rm mech}} = \frac{1-\beta}{C_0} - \frac{\alpha}{C_0} + \frac{1}{C},$$
(3)

Then, the characteristics of the revised circuit become the same as those of the conventional Mason's one, as listed in (i)–(iv) in the previous section, when observed from the M-source. Moreover, when the transducer is driven from the E-source in the electrically opened state (as in Fig. 2), the elastic stiffness is correctly regarded as $c^{\rm D}$, not $c^{\rm E}$, in the revised circuit model, which outperforms the structure and characteristics of the conventional Mason's circuit.