# 1P1-1

# Elastic wave propagation in two-dimensional phononic crystals with periodic arrays of viscoelastic inclusions

粘弾性散乱体の周期配列からなる 2 次元フォノニック結晶における弾性波伝播

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## 1. Introduction

For a few decades, theoretical and experimental researches on phononic crystals, which are artificial periodic composite materials made of substances with different elastic properties, have been actively conducted. In particular, in recent research called phonon engineering, application of phononic crystal to heat transport as well as elastic waves has been attracted attention.

Owing to the ease of manufacturing and processing, polymers such as polystyrene and epoxy resin are frequently used as materials constituting phononic crystals. However, in the study of conventional phononic crystals, the viscoelastic properties of the constituent substances have been largely ignored. Even when the constituent materials are handled as viscoelastic bodies, we have merely shown the effect of slightly modulating the characteristics of phononic crystals, which had been known hitherto. In the present work, we will construct a new elastic wave device using phononic crystal by positively introducing viscoelastic properties which have been neglected so far.

## 2. Model and methodology

Figure 1(a) illustrates the system considered here. The two-dimensional (2D) phononic crystal is composed of periodic arrays of epoxy circular inclusions embedded into Pb substrate. The finite number of the scatterers in the phononic crystal is $n$. The radius of the circular inclusion is $r_0$ and the period of the phononic crystal is $a$. The filling fraction is defined by $f=\pi r_0^2/a^2$. Epoxy resin is assumed to have the viscoelastic property which is expressed by the generalized Maxwell solid model as shown in Fig 2. The incident and detecting regions are composed of lead (Pb) without viscoelasticity. In all the regions, we assume that the thicknesses in the z-direction are infinite and homogeneous. As a result, we can deal with the system as a 2D one in the $x$ - $y$ plane.

In the present work, we deal with acoustic waves in viscoelastic materials like the method reported previously [1]. The equation of motion governing the displacement velocity $\boldsymbol{v}(t)$ is given by

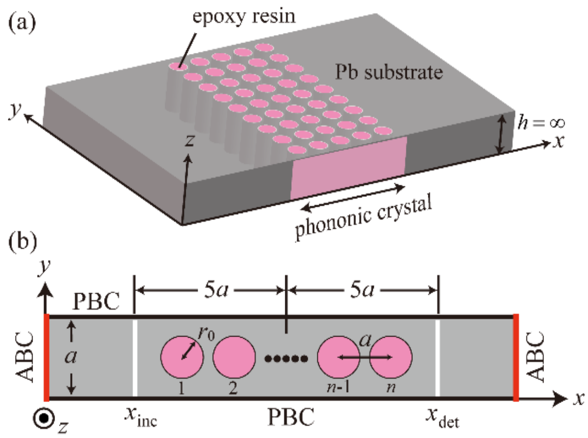


Fig. 1 (a) Schematic diagram of the system composed of three parts: the incident (left), phononic-crystal (med), and detecting (right) regions. The substrate and the circular inclusions are assumed to be lead (Pb) and epoxy resin, respectively. The $z$-direction has infinite thickness and homogeneity. (b) Unit cell used in the numerical calculation. ABC and PBC represent the absorbing and periodic boundary conditions, respectively. $x_{\mathrm{int}}$ and $x_{\mathrm{det}}$ are the positions where the wave is excited and the traveling wave is detected, respectively.

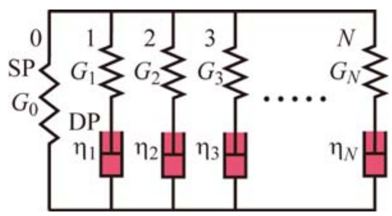


Fig. 2 $N$-element generalized Maxwell solid model. SP and DP represent a linear spring (spring constant $G$) and a dash pot. (viscosity η), respectively. The relaxation time is expressed by $\tau_G^{(j)}=\eta_j/G_j$ ($j$=1,2,3,…, $N$).

$$\frac{\partial v_i}{\partial t}=\frac{1}{\rho}\frac{\partial \sigma_{ji}(t)}{\partial x_j},$$

where $\rho$ is the mass density and $\sigma(t)$ is the stress tensor. The stress tensor is determined by the constitutive equation for the general linear viscoelastic fluid. Then, the total stress tensor is given by

$$\sigma(t) = 2\int_{-\infty}^{t} G(t-t')\mathbf{D}(t')dt' + \int_{-\infty}^{t}\left[K(t-t') - \frac{2}{3}G(t-t')\right]\left[\nabla\cdot\mathbf{v}(t')\right]\mathbf{I}dt'$$

where $G(t)$ and $K(t)$ are the steady shear and bulk relaxation moduli, respectively. **I** is the identity matrix of size 3 and $\mathbf{D}(t)$ is the rate-of-deformation tensor given by

$$\mathbf{D}(t) = \frac{1}{2}\left[(\nabla\mathbf{v}) + (\nabla\mathbf{v})^{T}\right].$$

We employ an $N$-element generalized Maxwell solid model in this work (See Fig. 2). When the relaxation times for $G(t)$ and $K(t)$ are set as $\tau_G^{(j)}$ and $\tau_K^{(j)}$, respectively, where $j = 1, 2, \cdots, N$ represents the set of a spring and a dashpot connected in series. Then, $G(t)$ and $K(t)$ are given by

$$\alpha(t) = \alpha_0 + \sum_{j=1}^{N}\alpha_j \exp\left[-t/\tau_\alpha^{(j)}\right] \quad (\alpha = K \text{ or } G).$$

We numerically solve these equations by a finite-difference time-domain (FDTD) method. Figure 1(b) indicates the computational domain, which has an incident region, a phononic crystal, and a detecting region, and is subject to periodic boundary conditions (PBCs) and absorbing boundary conditions (ABCs). $x_{\text{inc}}$ and $x_{\text{det}}$ respectively indicate the cross sections for the excitation and detection of acoustic waves, which are $5a$ apart from the origin set at the center of the system, as shown in Fig. 1(b). Applying an FDTD method to the unit cell, we can obtain the time evolutions of the velocity field and stress tensors at each spatial grid. Then, we can calculate the acoustic Poynting vector at $x_{\text{det}}$ in frequency domain. Using them, we can obtain the transmission rate as a function of frequency.

## 3. Numerical results and discussions

Figure 3 shows the transmission rates for an incident longitudinal wave as a function of frequency. The parameters describing the system are $r_0$=40 mm, $a$=120 mm, i.e. $f$=0.35. For simplicity, we use the generalized Maxwell solid model for $N$=1. Regarding the viscoelasticity of epoxy resin, we use a one-relaxation-time approximation: $\tau=\tau_K=\tau_G$, which we change from $10^{-6}$ to $10^{-2}$ sec in our calculation. We find that the bump of the transmittance between 130 and 140 kHz shows extreme attenuation as compared with the manner of attenuations of the bumps of the transmittance of the other region as the relaxation time decreases. The attenuation becomes extreme as the number $n$ of cylinders increases. This phenomenon is largely due to the polarization of the modes of elastic waves in that frequency region.

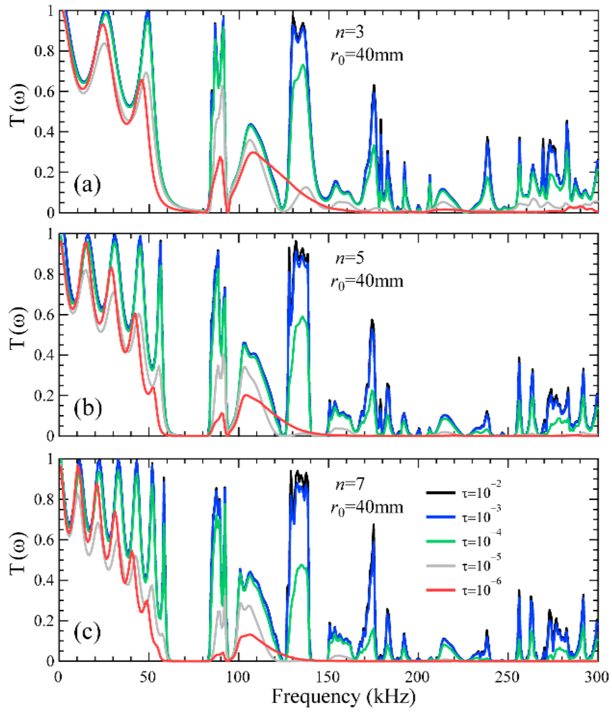


Fig. 3 Transmission rate vs frequency for (a) $n$=3, (b) $n$=5, and (c) $n$=7, respectively. The incident wave is longitudinal one. The colorful solid lines denote the differences of the relaxation time.

## 4. Conclusions

We have investigated the transmission characteristics of elastic waves through two-dimensional phononic crystals with periodic arrays of viscoelastic inclusions embedded into an elastically isotropic metal substrate. The inclusions that we assumed is expoxy resin, which can have a variety of viscoelastic properties by mixing various curing agent. We can positively utilize the viscoelasticity of polymers to the modulation of elastic wave properties in phononic crystals.

## Acknowledgment

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## References

1. Y. Tanaka, S. Tomioka, Y. Shimomura, and N. Nishiguchi: Jpn. J. Appl. Phys. **56** (2017) 034101.

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