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Analysis of the electromechanical characteristics of a piezoelectric multilayered structure for in-air ultrasound radiation

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1. Introduction

Ultrasonic transducers enable noncontact distance measurement based on pulse-echo method in air. Such ultrasonic transducers can be constructed in the form of a multilayered piezoelectric plate using a disk-shaped piezoelectric ceramic. At this time, the piezoelectric multilayered structure is a two-layered structure composed of a piezoelectric ceramic part generating a sound wave and a vibration plate part, or a three-layered structure having a backing layer added thereto [1, 2].

The most important factor in developing the sensor is to determine the resonant frequency. In this study, the vibration characteristics of the multilayered piezoelectric plate are investigated through theoretical analysis, and the resonant frequency is derived as an exact functional form of the structural variables of the plate. Furthermore, the sound pressure at the far field was calculated using the derived from and the beam pattern of the multilayered piezoelectric plate was calculated as well.

2. Model and Resonant Frequency Calculation of a Piezoelectric Multilayered Structure

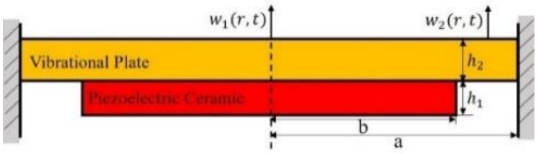

Fig. 1 Model of a two-layered piezoelectric structure.

To obtain the equation of bending vibration, we considered the kinetic energy, potential energy, bending moment and shear force of the multilayered structure and assumed that the radii are much larger than the thickness [3]. In order to simplify the derivation, the anisotropic stiffness of the piezoelectric ceramics was converted into Young's modulus and Poisson's ratio of an isotropic material of equivalent properties [4]. The adhesion of each layer is assumed perfect, and both ends of the vibrational plate are fixed as shown in **Fig. 1**.

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The z-axis is perpendicular to the vibration plane, and the neutral axis of the vibrating body is defined as the origin. First, the equation for the two-layered structure is set up. In this case, the axial displacement of the region where the piezoelectric ceramic is bonded to the vibrational plate is defined as $w_1(r,t) = w_1(r)e^{i\omega t}$ while that of the region where only the vibrational plate exists is defined as $w_2(r,t) = w_2(r)e^{i\omega t}$. Then, the equations of the motion are obtained as

$$(D_{21} + D_{22})\nabla^4 w_1 + (\rho_1 h_1 + \rho_2 h_2)\ddot{w}_1 = 0, \quad (0 \le r \le b) \tag{1}$$

$$D_{12}\nabla^4 w_2 + \rho_2 h_2 \ddot{w}_2 = 0, (b \le r \le a) \tag{2}$$

where D_{21} is flexural stiffness coefficient of the piezoelectric ceramic layer and D_{22} is flexural stiffness coefficient of the vibrational plate layer at the region where the piezoelectric ceramic is bonded to the vibrational plate while D_{12} is the flexural stiffness coefficient of the vibrational plate at the region without the piezoelectric plate. ρ_1 and h_1 are density and thickness of the piezoelectric ceramic, ρ_2 and h_2 are those of the vibrational plate, respectively, a is the radius of the vibrational plate, and b is the radius of the piezoelectric ceramic.

The solutions of Eqs. (1) and (2) are

$$w_1(r) = A_1 J_0(k'r) + A_2 I_0(k'r), \tag{3}$$

$$w_2(r) = A_3 J_0(kr) + A_4 Y_0(kr) + A_5 I_0(kr) + A_6 K_0(kr) \tag{4}$$

where A_1, A_2, A_3, A_4, A_5, and A_6 are amplitude constants, J_0, Y_0, I_0, and K_0 are the zero-order Bessel functions, $k^4 = \frac{\rho_2 h_2}{D_{12}}\omega^2$, $k'^4 = \frac{\rho_1 h_1 + \rho_2 h_2}{D_{21} + D_{22}}\omega^2$, and $\omega = 2\pi f_r$.

By applying the boundary conditions to Eqs. (3) and (4), the k is obtained and the resonant frequency can be obtained by using Eq. (5).

$$f_r = \frac{1}{2\pi}\sqrt{\frac{D_{32}}{\rho_2 h_2} k^4} \tag{5}$$

The three-layered structure is a structure in which a backing layer is added to the rear surface of the piezoelectric ceramic plate. The resonant frequency is obtained in the same manner as for the two-layered structure. The axial displacement is equal to the two-layered structure, and the equations of motion and solutions are similar to the two-layered structure. The resonant frequency can be obtained by using the same boundary conditions as those for the two-layered structure and using Eq. (5).

Further, the resonant frequency of the four-layered structure including a bonding layer between the can be calculated. The bonding layer is inserted in between the piezoceramic plate and the vibrational plate, and the resonant frequency is obtained in the same manner as for the two- and three-layered structures.

Fig 2 compares variation of the resonant frequency calculated with the Eq. (5) and that calculated with the finite element method, which shows excellent agreement. The vibrational plate is made of aluminum, the piezoelectric ceramic is PZT-5A, and the backing layer is sponge. The radii of the vibrational plate and piezoelectric ceramics are varied.

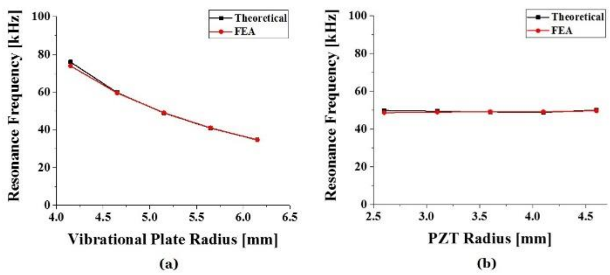

Fig. 2 (a) resonant frequency vs. vibrational plate radius, (b) resonant frequency vs. piezoelectric ceramic plate radius.

3. Analysis of the Beam Pattern of the Piezoelectric Multilayered Structure

The displacement $w_1(r,t)$ and $w_2(r,t)$ are obtained by substituting the resonant frequency into the solution of the equation of motion and calculating the amplitude constant. Then the velocity of the vibration is substituted into Eqs. (6) and (7) to find the sound pressure p from the multilayered structure that works as an ultrasonic transducer [5].

$$p_1(R,\theta,t) = -i\frac{\rho_a \omega}{2\pi R}\int_S u_1(r)e^{i(\omega t - k^* R - k^* r\cos\alpha\sin\theta)}dS, \quad (6)$$

$$p_2(R,\theta,t) = -i\frac{\rho_a \omega}{2\pi R}\int_S u_2(r)e^{i(\omega t - k^* R - k^* r\cos\alpha\sin\theta)}dS, \quad (7)$$

$$p = p_1 + p_2, \quad (8)$$

where R is the distance from the source, $k^* = \frac{\omega}{c}$, c is the sound velocity of air, and ρ_a is the density of air.

Fig. 3 compares the beam patterns obtained by using Eq. (8) and that obtained by replacing the $u(r)$'s in Eqs. (6) and (7) with constants.

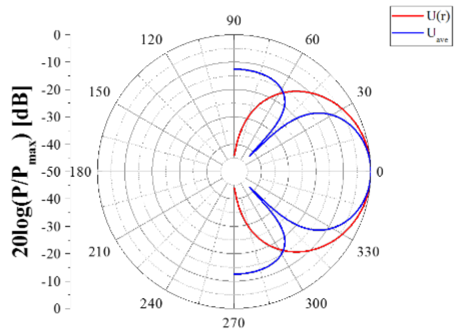

Fig. 3 Beam pattern of the four-layered piezoelectric structure.

4. Conclusions

We investigated the vibrational characteristics of the multilayered piezoelectric structure through theoretical analysis. Results of the theoretical analysis of the resonant frequency showed excellent agreement with those from the finite element analysis, which confirmed the validity of the theoretical analysis in this work. Beam patterns of the multilayered piezoelectric plate were also analyzed with the theoretical functions.

This study will contribute to more accurate and simple design of ultrasound transducers.

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