1. Introduction

It has been experimentally reported that the critical temperature difference for sound generation in a thermoacoustic engine with a wet stack is considerably smaller than that with a dry stack. Raspet and his coworkers have studied the effect of evaporation and condensation on a thermoacoustic engine based on the Eulerian point of view. The aim of the present study is to understand dynamical effect of evaporation and condensation based on the Lagrangian point of view.

There have already been some theoretical models of a thermoacoustic engine based on the Lagrangian point of view. The model of the present paper is unique because Rott equations are taken into account for the first time in such models based on the Lagrangian point of view.

2. Model

Translational motion as well as expansion and contraction of a fluid parcel is numerically simulated (Fig. 1). The amplitudes of acoustic pressure and particle velocity are calculated as a function of position along a narrow tube using Rott equations. The instantaneous acoustic pressure and particle velocity are calculated at a position of a fluid parcel using their amplitudes at the position. Using instantaneous particle velocity, translational motion of a fluid parcel is simply calculated as follows.

\[ x(t + \Delta t) = x(t) + u\Delta t \quad (1) \]

where \( x(t) \) is a position of a fluid parcel at time \( t \), \( \Delta t \) is a small time step, and \( u \) is instantaneous particle velocity at time \( t \) and position \( x \).

While number of air molecules is constant in a fluid parcel, number of water vapor molecules changes with time by evaporation and condensation. The instantaneous volume of a fluid parcel is determined by instantaneous pressure, temperature, and number of molecules inside a fluid parcel as follows.

\[ V(t) = n_r(t)R_GT(t)/p(t) \quad (2) \]

where \( V(t) \) is instantaneous volume of a fluid parcel, \( n_r(t) \) is instantaneous number of molecules of air and water vapor inside a fluid parcel in mol, \( R_G \) is the universal gas constant, \( T(t) \) is instantaneous temperature of a fluid parcel, and \( p(t) \) is instantaneous pressure inside a fluid parcel.

3. Results and Discussions

The results of numerical simulations for a traveling-wave thermoacoustic engine are shown in Fig. 2 (a) and (b) for a dry and a wet stack, respectively. Volume oscillation amplitude of a fluid parcel is increased by evaporation and condensation because evaporation (condensation) takes place at the expansion (contraction) of a fluid parcel. Accordingly, \( pV \) work done by a fluid parcel increases by evaporation and condensation. It implies that sound intensity increases by evaporation and condensation.

Another finding is the gradual shift of the mean position of a fluid parcel to higher temperature side, which is acoustic streaming. It
originates in Rott equations. However, detailed mechanism is very complicated, and will be studied in future in another paper.

On the other hand, volume oscillation amplitude decreases by evaporation and condensation for a standing-wave thermoacoustic engine. Nevertheless, slight presence of traveling-wave component results in the increase in \( pV \) work by evaporation and condensation. For the definition of traveling-wave component, see Ref. 8.

4. Conclusion

Under many conditions, volume oscillation amplitude of a fluid parcel is increased by evaporation and condensation in a wet stack of a thermoacoustic engine. Accordingly, \( pV \) work done by a fluid parcel increases by evaporation and condensation. It implies that sound intensity increases by the effect. The present model based on the Lagrangian point of view is nonlinear and results in acoustic streaming. The acoustic streaming originates in Rott equations. However, difference and similarity of the acoustic streaming to Gedeon and Rayleigh streaming are unclear at present, and will be studied in future.

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References