Analysis of Elastic Vortex Waves in Optical Fiber for Optical Vortex Mode Conversion

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1. Introduction

Optical vortex carrying Orbital Angular Momentum (OAM) has potential to increase capacity in the future optical network. For OAM multiplexing, it is necessary to generate and handle plural OAM modes. Elastic vortex waves along an optical fiber can be used for mode conversion between OAM modes through acousto-optic (AO) interaction. [1-4] In this study, we discuss elastic vortex waves and the frequency required for the AO mode conversion between OAM modes.

2. OAM mode conversion by elastic wave

According to previous study, the interaction between elastic waves and optical waves causes optical mode conversion in an optical fiber. This study, we consider vortex waves formed from orthogonal flexural waves as shown in Fig.1. We consider a graded index fiber as an optical fiber. The incident OAM mode is converted to the other OAM mode along the fiber.

3. Flexural wave dispersion relation

We analyze flexural wave in the cylindrical waveguide to obtain the dispersion relation. The displacement is expressed by equations (1) - (3).

\[
\begin{align*}
u_r &= [4k_x J'_\nu (k_r r) + B k_x J'_\nu (k_r s) + C J_\nu (k_r r)] \left[ \begin{array}{c} \sin(n\phi) \\ \cos(n\phi) \end{array} \right] e^{i(n-a)z} \quad (1) \\
u_\phi &= [4n \nu_\phi (k_r r) + B k_x J'_\nu (k_r s) + C k_x J'_\nu (k_r r)] \left[ \begin{array}{c} \cos(n\phi) \\ -\sin(n\phi) \end{array} \right] e^{i(n-a)z} \quad (2) \\
u_z &= -i [4 k_x J'_\nu (k_r r) + B k_x J'_\nu (k_r s)] \left[ \begin{array}{c} \sin(n\phi) \\ \cos(n\phi) \end{array} \right] e^{i(n-a)z} \quad (3)
\end{align*}
\]

Here, \( \omega \) is the angular frequency, \( r \) is the cylindrical waveguide radial length, \( k_\nu \) is the propagation constant in the \( z \) direction, and \( J_\nu \) is the Bessel function of the first kind and of order \( n \). \( J'_\nu \) denotes its derivative with respect to the argument, and \( k_d \) and \( k_i \) are given by

\[
k_d^2 = \frac{\omega^2}{c_d^2} - k_\nu^2 \quad (4)
k_i^2 = \frac{\omega^2}{c_i^2} - k_\nu^2 \quad (5)
\]

where the bulk dilatational and transverse wave velocities \( c_d \) and \( c_i \) are given by the density \( \rho = 2.20 \times 10^3 \text{ kg/m}^3 \), Lame’s constants \( \lambda = 1.6 \times 10^{10} \), and \( \mu = 3.12 \times 10^{10} \) through

\[
c_d^2 = \left( \frac{\lambda + 2\mu}{\rho} \right)
c_i^2 = \frac{\mu}{\rho} \quad (6)
\]

So, the following determinant (8) is given from the boundary conditions for the stress tensor.

\[
\begin{vmatrix}
\frac{\rho - 1}{\rho} q_1 (s-1) & n^2 - 1 - q_1 (2n-1) \\
2n^2 - 1 & 2n^2 - 2q_1 (s-1) - q_1 (2n-1) \\
\frac{\gamma_n (q_1) - n}{\gamma_n (q_1) - n} & 2n^2 - 2q_1 (s-1) - q_1 (2n-1) \\
\end{vmatrix} = 0 \quad (8)
\]

Here, \( \gamma_n (q) = q J_{n+1} (q) / J_n (q) \) and \( x = c^2 / 2c_d^2 \). The dispersion relation is obtained as shown in Fig.2.

Fig. 1 OAM mode conversion by elastic vortex waves

Fig. 2 Mode dispersion relation of flexural mode
4. Phase matching condition

From the phase matching condition for the mode coupling between the optical waves and elastic waves, and dispersion relation of the elastic waves, the frequency that mode coupling occurs is obtained. The phase matching condition is given by

\[ \beta(\omega_i) - \beta(\omega_p) = k. \]  

Here, \( \beta \) is the propagation constant of optical wave. \( k \) is the propagation constant of elastic wave. From this equation and Fig.2, we can identify elastic frequency that executes mode conversion of optical mode. We consider a graded index fiber. Then, the propagation constant is given by

\[ \beta = \sqrt{\beta_0^2 n^2(0) - 2 \beta_0 n(0) g(2m + \nu + 1)}. \]  

Here, \( \beta_0 \) is the optical propagation constant in vacuum. \( n(0) \) is the refractive index of center of the fiber and \( g \) is the focusing constant. \( m \) is the vortex rotation order and \( \nu \) is the number of nodes in radial direction. As an example, we consider \( m=0 \rightarrow 1 \) and \( \nu =1 \). The optical wavelength is 1550nm and we assume \( n(0)=1.46 \). The graph obtained from equation (9) is Fig.3. The intersection points of Fig.3 are about 1.2MHz and 0.80MHz.

5. Elastic vortex wave

Based on the equation of displacement, we confirmed whether the flexural wave is a vortex wave by making graphs of intensity and phase. It can be confirmed that the flexural wave is the vortex wave when synthesized by orthogonalizing x-axis component and y-axis component of displacement with phase difference of \( \frac{\pi}{2} \).

6. Conclusion

In this study, we found that the ultrasonic wave frequency for mode coupling are 1.2MHz or 0.80MHz. We showed a vortex elastic wave formed from flexural waves. As a future plan, we investigate AO coupling efficiency for OAM mode conversion.

References