A Comparison Examination on Treatment of Interface between Different Media for Sound Field Simulation Using CIP Method

1. Introduction

To date, as a result of advances in computer technology, numerical simulation for sound wave propagation has been frequently investigated. For sound field simulation, the development of accurate and effective analysis method in time domain has become an important technical issue.

The constrained interpolation profile (CIP) method, a novel low-dispersive numerical scheme, is a kind of method of characteristics (MOC) employing a co-located grid system. In our previous study, we have applied the CIP method to solving the linearized sound propagation equation [1-3].

In sound filed simulation using CIP method setting boundary interface between different media or material is required. That is, it is necessary to implement the reflection and transmission condition for sound fields and their spatial derivatives on an interface[3].

In this study, we implemented treatment of interface between different media in CIP sound field analyses and show the simulation result using CIP method. In addition, we compared the CIP method with Finite-difference time-domain (FDTD) method employing a staggered grid system.

2. Numerical Analysis Using CIP Method

Linearized governing equations of sound field are given in Eq. (1) and Eq. (2).

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla \cdot \mathbf{p} \quad (1)
\]

\[
\nabla \cdot \mathbf{v} = -\frac{1}{K} \frac{\partial p}{\partial t} \quad (2)
\]

In these equations, \( \rho \) denotes the density of the medium, \( K \) is the bulk modulus, \( p \) is the sound pressure, and \( \mathbf{v} \) is the particle velocity.

Here, assuming the analysis of sound field propagation of the \( x \)-direction, we can obtain the following equations from Eq. (1) and Eq. (2).

\[
\frac{\partial}{\partial t} p + c \frac{\partial}{\partial x} Z v_x = 0 \quad (3)
\]

In these equations, \( Z \) indicates the characteristic impedance (i.e. \( Z = \sqrt{K \rho} \)) and \( c \) represents the sound velocity in the medium (i.e. \( c = \sqrt{K / \rho} \)).

Then, by addition and subtraction of Eq. (3) and Eq. (4), the advection equations are given as

\[
\frac{\partial}{\partial t} (p \pm Z v_x) \pm c \frac{\partial}{\partial x} (p \pm Z v_x) = 0 \quad (5)
\]

In the CIP method, simple spatial differentiation of Eq. (5), the advection equations of their derivatives are also obtained as the following equations.

\[
\frac{\partial}{\partial t} (\partial_x p \pm Z \partial_x v_x) \pm c \frac{\partial}{\partial x} (\partial_x p \pm Z \partial_x v_x) = 0. \quad (6)
\]

3. Treatment of Interface between different media

Next, we present treatment of interface between different media. We assume that the interface between the medium 1 (\( \rho_1, K_1 \)) and the medium 2 (\( \rho_2, K_2 \)) is \( x = i_0 \Delta x \) and that their impedance are respectively

\[
Z_1 = Z(i_0 - 1/2), \quad (9)
\]

\[
Z_2 = Z(i_0 + 1/2). \quad (10)
\]

Then, from Eqs. (7) and (10), we can obtain the interface conditions of \( F_{x \pm} \) and \( G_{x \pm} \) as

\[
F_{x \pm}^{n}(i_0) = T_{12} F_{x \pm}^{n}(i_0) + T_{21} G_{x \pm}^{n}(i_0) \quad (11)
\]
\[
F_{x1}^n (i_0) = T_{12} F_{x1}^n (i_0) + T_{21} F_{x2}^n (i_0) \quad (12)
\]
\[
G_{x2}^n (i_0) = T'_{21} G_{x1}^n (i_0) + T'_{12} G_{x2}^n (i_0) \quad (13)
\]
\[
G_{x1}^n (i_0) = T'_{12} G_{x1}^n (i_0) + T'_{21} G_{x2}^n (i_0) \quad (14)
\]

In these equations,
\[
T_{12} = \frac{2Z_2}{Z_1 + Z_2}, \quad \Gamma_{12} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad (15)
\]
\[
T_{21} = \frac{2Z_1}{Z_1 + Z_2}, \quad \Gamma_{21} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \quad (16)
\]
\[
T'_{12} = \frac{c_1}{c_2} T_{12}, \quad T'_{21} = \frac{c_1}{c_2} T'_{21} \quad (17)
\]

Here, \( T \) and \( \Gamma \) are the transmission coefficients and reflection coefficients, respectively. In the CIP method, the interface conditions between different media are defined as above equations. Hence, respective procedure in the \( y \)-direction as well as in the \( x \)-direction can be applied.

4. Computational results

Calculation parameters used in the calculations are summarized in Table 1. We compared simulation results using the CIP method and the FDTD method[4] by changing \( Z \) and \( c \) of Media 2.

Figures 2 shows the amplitude ratio of the input wave and reflected wave against points per wavelength (PPW) at each angle. Results by the CIP method, the FDTD method and theoretical value are depicted in these figures. In Fig. 2 (a), we assume \( \Gamma_{12} = 0.5 \) \((Z_2=3Z_1)\) and \( c_2 = 1/3c_1 \). We can ascertain the reflected waves from the interface in both figures.

Figures 2 shows the sound pressure distribution obtained by the CIP method and The FDTD method at \( n = 2000 \) time step. In this computation, we assume \( \Gamma_{12} = 0.8 \) \((Z_2=9Z_1)\) and \( c_2 = 1/3c_1 \). We also assume that \( \Gamma_{12} = 0.8 \) \((Z_2=9Z_1)\) and \( c_2 = c_1 \) in (b), and that \( \Gamma_{12} = 0.5 \) \((Z_2=3Z_1)\) and \( c_2 = 1/3c_1 \) in (c).

From Fig. 2 (b), we can ascertain results of both the CIP method and FDTD method become different from the theoretical value as the angle is large. In Fig. 2 (a) and (c), it is clarified that the error of the CIP method is smaller than that of the FDTD method. On the other hand, we can see the difference between calculated results and theoretical value as reflection coefficient is larger.

5. Conclusion

We examined the treatment of interface between different media for sound field simulation using CIP method. We apply the conditions to 2-dimmensional computation of sound fields and evaluate each interface condition.

References