# Wave-propagation properties of alpha and beta Lanthanum studied by ab-initio calculations

第一原理計算を用いたアルファおよびベータ La の超音波伝播 特性

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## 1. Introduction

Lanthanum (La) is the first metal of the lanthanoid, which includes Ce, Pr, Nd, Yb, and so on. La exhibits the double hexagonal closed packed (dhcp) structure ( $\alpha$ -La) below 600 K, the face centered cubic (fcc) structure ( $\beta$ -La) between 600 and 1140 K, and the body centered cubic (bcc) structure ( $\gamma$ -La), as shown in Fig. 1. Since the energy of  $\alpha$ -La and  $\beta$ -La is close,  $\beta$ -La can exist below 600 K as a meta-stable phase.  $^{\left[ 1,\,2\right] }$  The dhcp structure consists of ABAC... layers the along [0001] direction, whereas the fcc structure consists of ABC... layers along the [111] direction. Stassis et al. measured the elastic constants of monocrystalline  $\beta$ -La by the neutron scattering method.<sup>[3]</sup> However, those of  $\alpha$ -La have never been measured. Thus, acoustic properties of  $\alpha$ -La remain unclear although Lanthanum composites are widely used in many applications such as LaNi5 for hydrogen automobiles,  $LaB_6$  for electronic microscopes, La<sub>2</sub>O<sub>3</sub> for optics for infrared absorbing glass.

Electronic structures of La largely affect its physical properties. La has two 6s electrons, one 5d electron, and no f electrons. It also has a high density of state at the Fermi level. These feathers lead to unusual physical properties such as relatively high superconductive transition temperature of 5–6 K at ambient pressure,<sup>[4]</sup> which reaches 13 K at 20 GPa.<sup>[5]</sup> The pressure dependence of sound velocities also shows anomalies,<sup>[6]</sup> where

C-layer A-layer B-layer A-layer (a) (b)

Fig. 1 The structures of (a) dhcp  $\alpha$ -La and (b) fcc  $\beta$ -La.

the contribution of the electronic structure is of interest due to a strong electron-phonon coupling and *s*-*d* band transition. For example, elastic constants of Pd and Pt show unusual temperature dependence due to 4d electrons.<sup>[7,8]</sup>

In this study, we calculate the elastic constants and sound velocities of  $\alpha$ -La and  $\beta$ -La based on the density functional theory, and discuss wave-propagation properties, anisotropy, and their pressure dependence.

# 2. Calculation

We used the Vienna *ab-initio* simulation package (VASP), using several types of generalized gradient approximation (GGA-PBE)<sup>[9, 10]</sup> potentials and local density approximation (LDA)<sup>[11]</sup> potentials. The cutoff energy of plane wave function is 1300 eV. We applied the Methfessel-Paxton scheme with the width of 1 $\mu$ -1 eV for the partial occupation of electrons, and the *k*-point mesh is from  $8 \times 8 \times 8$  to  $40 \times 40 \times 40$  made by the Monkhorst pack.

First, we calculate the total energy and volume for each calculation condition, and determine the lattice constants. The elastic constants are calculated through the strain-energy relationships by applying  $\pm 1\%$  strains for several deformation modes. We also change the volume to calculate the pressure dependence of the elastic constants.

#### 3. Result

First, we calculated the lattice constants a and



Fig. 2 The difference between the reported and calculated lattice constants a and c of  $\alpha$ -La using several potentials.



Fig. 3 The difference between the reported and calculated elastic constants of  $\alpha$ -La using several potentials.

*c* of  $\alpha$ -La using GGA and LDA potentials with a 10  $\times$  10  $\times$  10 *k*-point mesh and 0.1-eV width smearing. All of the GGA potentials provide good agreement with measurement values<sup>[12]</sup> whereas LDA potentials underestimate them by ~5% as shown in **Fig. 2**. Elastic constants calculated by GGA also agree with measurement values<sup>[13]</sup> within 5% as shown in **Fig. 3**. On the other hand, LDA fail to provide proper results.

Second, we calculate the elastic constants of  $\alpha$ -La using a GGA, and evaluate the anisotropy. Ranganathan and Starzewski defined the universal elastic anisotropy index  $A^U = 5G_V/G_R + B_V/B_R - 6$  (>0), where G and B are the shear and bulk moduli, and subscribes of V and R mean Voigt and Reuss approximations, respectively.<sup>[14]</sup> We plot  $A^U$  of several hexagonal materials for the c/a ratio in Fig. 4. Here, La, Nd, and Pr are plotted for c/2a since they show the dhcp structure. We find that  $\alpha$ -La exhibits relatively large anisotropy among hexagonal materials. (Note that, Cd, Zn, and Tl has much larger  $A^U$  of 1–2.)

The large anisotropy of  $\alpha$ -La is also represented by a polar velocity curve. We calculate the phase velocities in the *x*-*z* plane of  $\alpha$ -La ( $A^U$  =0.16) and Mg ( $A^U$  =0.04) as shown in **Fig. 5**. The two shear velocities propagating [101] direction of Mg show 8.4% difference, whereas those of  $\alpha$ -La show 17% difference. The difference between the fastest and slowest longitudinal velocities in the *x*-*z* plane of Mg and  $\alpha$ -La are 4.7 and 11%, respectively.

#### 4. Conclusion



Out-of-plane and in-plane distances ratio c/a or c/2aFig. 4 The universal anisotropy index  $A^U$  of hexagonal materials.



Fig. 5 Polar velocity curves in the x-z plane of (a)  $\alpha$ -La and (b) Mg.

We calculated the lattice constants and elastic constants of  $\alpha$ -La by the *ab-initio* method, and found that GGA provides proper values. Our calculation results indicate that  $\alpha$ -La exhibits relatively large velocity anisotropy among hexagonal materials. We will discuss its pressure dependence in further calculations.

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