Computational cost reduction by avoiding ray-link iteration of bent-ray method for sound speed image reconstruction in ultrasound computed tomography

反復演算を不要とする屈折経路推定法を用いた超音波トモグラフィー音速再構成の高速化

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1. Introduction

Ultrasound computed tomography (USCT) is a promising breast cancer diagnostic technique under investigation. It can provide three kinds of 3 dimensional (3D) images: sound speed, attenuation and reflection ^[1-4]. This study concerns the sound speed image, since it is able to give better information for quantitative diagnosis.

The previous methods for sound speed reconstruction can be classified into two categories: waveform-based methods [1] and ray-based methods ^[2-4]. The former is more promising, since it is based on the full wave equation. However, its procedure is complicated, and its computation is cumbersome. On the other hand, the ray-based method is simpler and more popular. There are two kinds of ray-based methods: straight-ray method (SRM) and bent-ray method (BRM). The SRM assumes straight-ray and neglects refraction, and the BRM traces bent-ray for each pair of emitter and receiver considering refraction. Consequently, the BRM gives better image quality than the SRM does, but higher computational cost. One important reason for the higher cost is that previous BRM iteratively traces a large number of bent-rays to find the bent-ray linking emitter and receiver, which is known as ray linking iteration.

In this study, a novel technique is proposed to reduce the computational cost of previous BRM by avoiding ray linking iteration. The proposed method successfully reduces the computational cost without reducing the reconstruction quality

2. Method

The BRM is a widely used method for sound speed reconstruction. It iteratively finds sound rays linking each pair of emitter and receiver, then corrects sound speed along these rays. In BRM, there are three steps for one iteration procedure ^[3]. First, first arrival time of transmissive wave is picked up as the travel time. Second, ray path between each pair of emitter and receiver is find using Fermat's

principle, which is expressed as:

$$r(s + \Delta s) = r(s) + \frac{dr}{ds}\Delta s + \frac{1}{2n} \Big[\nabla n - \Big(\nabla n \cdot \frac{dr}{ds} \Big) \frac{dr}{ds} \Big] (\Delta s)^2$$
(1)

where *n* denotes the refractive index, r and *s* are the position vector and the traveling distance from emitter, respectively. $r(s + \Delta s)$ and r(s) indicate the next and current positions. Finally, sound speed image is reconstructed using simultaneous algebraic reconstruction technique (SART).



(a) (b) Fig. 1. Ray linking. (a) ray-tracing, (b) function of lauch angle (θ) and arrival position (α). α_i and α_j are polar angle for emitter i and receiver j.

Ray linking is a technique to find the bent-ray linking a pair of emitter and receiver. it can be mathematically formulated as a boundary-value problem (BVP) because both emitter and receiver positions are given. BVP can't be directly solved using Eq. (1), since the laruch angle is not given. As shown in Fig. 1, the previous ray linking methods treat the arrival position (α) as a function (Fig. 1(b)) of the launch angle (θ) and try to find the launch angle, which is the same as finding the intersection of dotted line in Fig. 1(b). They suggested to use iteration method, such as bisection method, newton's method and so on. However, all these methods must iteratively trace multiple bent-rays for linking an emitter to a receiver, which is computationally expensive.

To avoid the cumbersome ray linking, a virtual receiver technique is proposed in this study. For this technique, the emitter position is fixed at the actual emitter position and the launch angle is set as the angle of straight line which connects the emitter to the receiver. The bent-ray is traced step by step using Eq.(1) until it arrivaes at the ring transdcuer. Then the arrival position is recorded as the position of virtual receiver. The travel time of the virtual receiver is interpolated from that of the actual receiver. Fig. 2 shows the schematic diagram of virtual receiver technique.



Fig. 2. Virtual receiver technique.

3. Results

Simulation, phantom and ex-vivo experiments had been conducted for validation.



Fig. 3. Simulation results. (a) simulation model, (b) and (c) are reconstructed sound speed images by previoius and proposed methods, respectively

Fig. 3 shows the simulation results. Fig. 3(a) is the simulation model which is a sound speed distribution map. Fig. 3(b) and (c) are reconstructed images by the previous actural receiver bent-ray method (ARBRM) and the proposed virtual receiver bent-ray method (VRBRM), respectively. They are very close to each other. Fig. 4 compares their center profiles.



Fig. 4. Center profiles of simulatino results.



Fig. 5. Phantom experiment results. (a) setup, (b) speed image, (c) center profiles comparison.

Fig. 5 shows the phantom experiment results. Fig. 5 (a) shows the setup of the phantom scanning. Then, Fig. 5(b) shows the reconstructed sound speed image by the VRBRM. Fig. 5(c) compares the center profiles of speed images reconstructed by the previous ARBRM and the proposed VRBRM, respectively.



Fig. 6 ex-vivo experiment results. (a) setup, (b) speed image, (c) center profiles comparison.

Fig. 6 shows the ex-vivo expriment results. Fig. 6(a) gives the scanning setup of ex-vivo experiment. Fig. 6(b) presents the reconstructed sound speed image by the proposed VRBRM. The center profiles of sound speed images reconstructed by the previous ARBRM and the proposed VRBRM are compared. Their center profiles nearly overlape each other, which demonstrates they are very close.

 Table1 Ray-tracing number and time cost. AR: the previous ARBRM; VR: the proposed VRBRM

	Ray trace Num.		Time cost (s)	
	AR	VR	AR	VR
Simulation	33×10^{4}	6.5×10 ⁴	385	125
Phantom	37×10^{4}		449	136
Ex-vivo	34×10^{4}		417	127

Compared to prevous ARBRM, the proposed VRBRM dramatically redues computational cost by avoiding ray linking and decreasing the number of ray-tracing. As Table 1 shown, the number of ray-tracing was dramatically decreased to about 20% and the reconstruction time was reduced to about 30%. Both the ARBRM and VRBRM were implemented by Matlab code, and excuted on a PC with Intel(R) Xeon(R) Processor E5640 2.66 GHz.

4. Conclusion

In this study, a novel technique is proposed to reduce the computational cost of the previous bentray reconstruction method by avoiding ray linking iteration and reducing ray-tracing number.

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