

## Experimental study on super-resolution imaging using vibro-Doppler measurement

バイブドロップラ計測による超解像イメージングに関する実験的検討

Takashi Miwa<sup>†</sup> and Seiya Kobayashi (Grad. School of Science and Tech., Gunma Univ.)  
三輪 空司<sup>†</sup>, 小林 誠也 (群馬大院 理工学府)

### 1. Introduction

Generally, the cross-range resolution in coherent imaging is restricted by a point spread function (PSF) of the sensor. Due to the lowpass characteristics of PSF in a spatial-frequency domain, we cannot obtain the high spatial-frequency information. In order to improve the cross range resolution, the bandwidth should widen in the spatial frequency domain. However, it is difficult because the spatial bandwidth is determined by the aperture of the sensor and frequency. However we can obtain the high wave number information with the same sensor if we can shift the wave number spectrum of a reflection coefficient distribution of the ROI. We have proposed a method in which the spatial frequency shift by using a vibro Doppler measurement. In this report, we experimentally demonstrate the super-resolution method with images obtained in vibro Doppler measurements.

### 2. Vibro-Doppler measurement

In this method, we use two kinds of waves. One is a sensing wave which gives a coherent-wave image of a ROI by scanning a transmitter and a receiver. Another is a vibration wave which gives Doppler Effect to a reflected wave. If a plane vibration wave having the frequency  $f_v$  is induced to the ROI, the reflector fluctuates. The displacement  $\xi_z(t, x)$  is represented with the wave number  $k_v$  of the vibration wave which propagates to the  $x$ -axis as

$$\xi_z(t, x) = \delta \cos(2\pi f_v t - k_v x) \quad (1)$$

where  $\delta$  is displacement of the vibration wave. Assuming that the sensing wave propagates only to the  $z$ -axis with no attenuation for simplicity, complex Doppler signals are obtained by convolution between a PSF  $w(x)$  and a product of a complex reflection coefficient distribution  $\dot{\gamma}(x)$  and the vibration wave as

$$\dot{g}(t, x) = \int w(x-x')\dot{\gamma}(x')\exp\{-2jk_u\xi_z(t, x')\}dx' \quad (2)$$

where  $k_u$  is a wave number of the sensing wave. Generally,  $\theta = k_u\delta$  is much smaller than the unity, the approximation  $\exp(j\theta) \cong 1 + j\theta$  can be applied.

Then, eq. (2) can be approximated as

$$\dot{g}(t, x) \cong \int w(x-x')\dot{\gamma}(x')\{1 - 2jk_u\xi_z(t, x')\}dx'. \quad (3)$$

By extracting frequency components at  $f = 0, \pm f_v$  of eq. (3), we obtain an original(static) image and vibro-Doppler images at the frequencies as follows:

$$\dot{g}_0(x) = \int w(x-x')\dot{\gamma}(x')dx' \quad \text{and} \quad (4)$$

$$\dot{g}_{\pm f_v}(x) = -j\theta e^{\pm j\theta} \int w(x-x')\dot{\gamma}(x')\exp(\mp jk_v x')dx'. \quad (5)$$

Here, the wave number spectrum in the  $x$ -axis are obtained

$$G_0(k_x) = W(k_x)\Gamma(k_x) \quad \text{and} \quad (6)$$

$$G_{\pm f_v}(k_x) = \mp j\theta W(k_x)\Gamma(k_x \pm k_v). \quad (7)$$

The high spatial frequency information of the statics signal as shown in eq. (6) is limited by low pass characteristics  $W(k_x)$  of the sensor. On the other hand, the wave number spectrum as shown in eq.(7), which is also limited by  $W(k_x)$ , have high spatial frequency information because the spatial frequency is shifted by Doppler effect of the vibration wave. We call this measurement as vibro-Doppler measurement [1].

### 3. SFCW vibro-Doppler system

**Figure 1** shows a block diagram of the vibro-Doppler measurement system. We use an ultrasonic(US) wave as the sensing wave. The reflector is a thin metallic wire having the diameter of 0.6 mm which is immersed in an agar gel 10 mm below the surface. The agar gel and US transducers are in a water. The original(static) image is obtained by scanning the US transducers to the  $x$ -axis. The axial ( $z$ -axis) resolution is obtained by a network analyzer having a center frequency 5 MHz and the bandwidth of 500 Hz. In the vibro-Doppler measurement, we selected the vibration frequency  $f_v$  to be 500 Hz. In order to obtain Doppler image, a transmitting signal from the network analyzer is modulated with a vibration frequency  $f_v$  by an image cancelling mixers, resulting that the frequency of the transmitting wave is  $f_u - f_v$ . The vibrating wire modulates the reflected wave,

resulting in the frequency of  $f_u$  and  $f_u - 2f_v$ . So, we can obtain a  $f_v$  vibro-Doppler image in the network analyzer, which spatially modulates the sinusoidal spatial variation to the static image [2].

#### 4. Measurement results and discussion

We discuss a 1D profile in the scanning direction which is extracted from the 2D image at the depth of the wire. We set two wires separated by 2 mm. The cross-range resolution of the US transducer is less than 1 mm. We use this image as a true image of the ROI. In order to experimentally demonstrate the proposed super-resolution algorithm, we apply it to a pseudo-experimental data which is made by filtering the true image with a narrow-band spatial filter and by adding a white noise. **Figure 2** shows a wave-number spectrum of a pseudo-experimental vibro-Doppler signal. The assumed SNR is 40 dB.

We can roughly estimate the wave number  $k_v$  of the vibration wave in the agar. In order to improve the cross range resolution, the spatial-frequency spectrum of the vibro-Doppler signal should be shifted back to the original wave number by  $k_v$ . We can also roughly estimate the displacement  $\delta$  of the reflector by vibration. Then, we can compensate the amplitude degradation by  $\theta = k_u \delta$ . This is called as a spatial demodulation process. The demodulated spatial frequency spectrum of vibro-Doppler images is shown as **Figure 3**. Because the amplitude of the vibro-Doppler image is 20 dB less than that of the original one, the noise level increases due to demodulation process. By using these signals, the bandwidth expansion is achieved. The super-resolution image  $\hat{g}_{SR}(x)$  is given with compensation factor  $C = \theta e^{j\phi}$  by

$$\hat{g}_{SR}(x) = \hat{g}_0(x) + C\hat{g}_{-f_v}(x)e^{jk_v x} - C\hat{g}_{+f_v}(x)e^{-jk_v x} \quad (8)$$

**Figure 4** shows a super-resolution result by vibro Doppler measurement. In the original image, the two wire is not separated. However. In the proposed image, we can clearly find two reflections from the two wires. The position and reflection coefficient of the original image almost correspond to that of the true image. The bandwidth of the proposed method is 2.8 times wider than that of the original image.

#### 5. Conclusion

In this report, we experimentally demonstrate the super-resolution method with vibro Doppler measurement. The vibro Doppler measurement can measure high spatial frequency information. Moreover, it can be applied to super-resolution imaging. However, a disadvantage of this method is

SNR degradation. So, the vibro-Doppler image should be measured higher SNR than the static image.

#### Acknowledgement

This work was supported by Grant-in-Aid for Challenging Exploratory Research 26630179.

#### References

1. T. Miwa et al: Proc. of 33th USE, 185 (2012).
2. A. Kaneko et al: Proc. of 35th USE, 59 (2014).

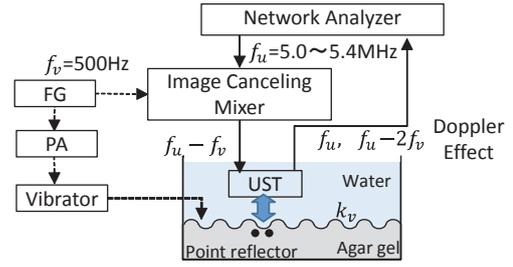


Fig. 1 SFCW vibro-Doppler measurement system

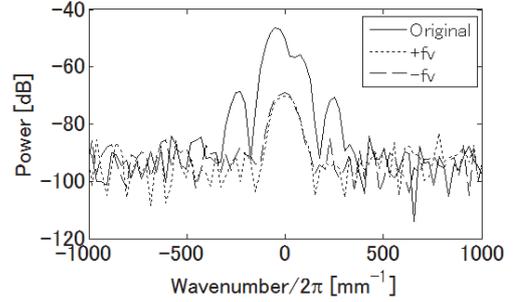


Fig. 2 Wavenumber spectrum obtained in vibro-Doppler measurement (SNR:40dB)

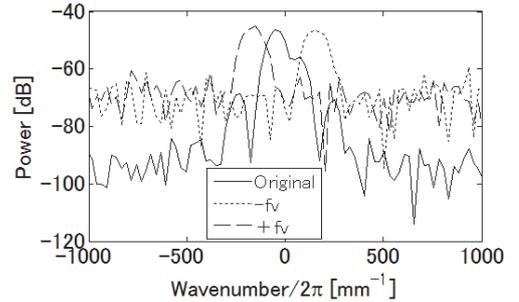


Fig. 3 Wavenumber spectrum after spatial demodulation

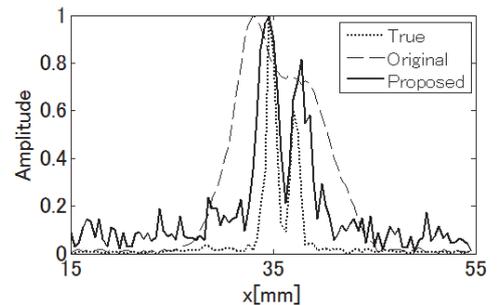


Fig. 4 Super-resolution image  $\hat{g}_{SR}(x)$  for two point reflectors separated by 2 mm