

# Propagation Characteristics of Love-Type Wave in a Structure with Viscoelastic and Elastic Layers

粘弾性-弾性構造における Love 波の伝搬特性

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## 1. Introduction

Photopolymer, a resin cured by irradiation of visible or ultraviolet light, is used in wide range of manufacturing processes, such as nanoimprint for semiconductor fabrication<sup>1)</sup>, and liquid crystal display coating for protection and improvement of physical and chemical functions<sup>2)</sup>. On the other hand, physical properties of a photopolymer, viscoelasticity, changes dramatically in curing process. Therefore, viscoelasticity evaluation of the photopolymer is important to prevent insufficient curing, and to optimise manufacturing process using the photopolymer.

Evaluation of material viscoelasticity using surface acoustic wave (SAW), which propagates along the surface of piezoelectric substrate (called shear-horizontal SAW), has been proposed by Morita *et al*<sup>3)</sup>. However, the shear-horizontal SAW only propagates along the piezoelectric substrate. Hence, the viscoelastic materials should be spread on the piezoelectric substrate, which results in bringing the piezoelectric substrate into contact with the viscoelastic material. In this study, viscoelasticity evaluation without bringing the piezoelectric substrate into contact with the viscoelastic material is proposed. **Figure 1** shows previous and proposed method for viscoelasticity evaluation. Previous method using shear-horizontal SAW (Fig. 1(a)) requires to evaluate viscoelasticity on the piezoelectric substrate. Meanwhile, proposed method using Love-type wave (Fig. 1(b)) dose not require the piezoelectric substrate. As pilot study, propagation characteristics of Love-type wave along the viscoelastic surface layer are simulated by changing viscoelastic parameter, and capability for viscoelasticity evaluation is discussed.

## 2. Theory

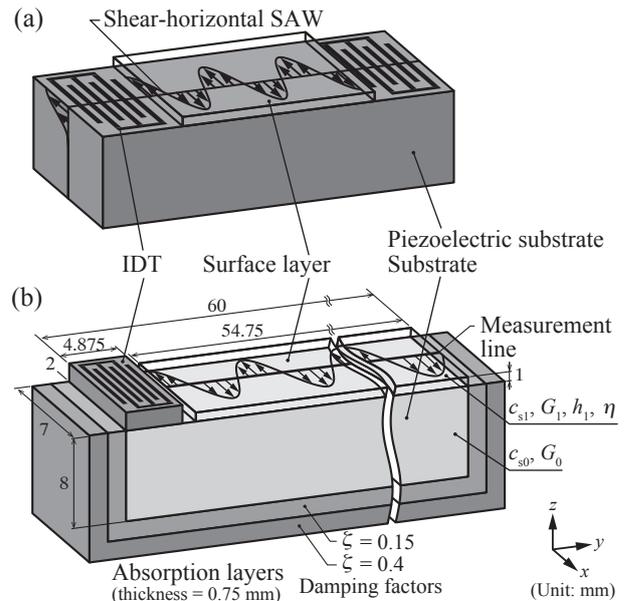
### 2.1 Characteristic equation of Love wave

Love wave propagates along the thickness-finite surface layer on half-space substrate under condition that shear wave velocity of the surface layer,  $c_{S1}$ , is less than that of the substrate,  $c_{S0}$ . Therefore, the wave which propagates along the surface layer and thickness-finite substrate as shown in Fig. 1(b), is distinguished as Love-type wave in this paper.

Phase velocity of Love wave is given theoretically. Assuming both the surface layer and the substrate as

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**Fig. 1** Comparison between (a) previous (using shear-horizontal SAW) and (b) proposed method (using Love-type wave) for viscoelasticity evaluation. The (b) also shows as simulation model of Love-type wave propagation for finite element method.

elastic materials, characteristic equation of Love wave becomes<sup>4)</sup>,

$$\tan \left( \sqrt{\frac{c^2}{c_{S1}^2} - 1} kh_1 \right) = \frac{G_0 \sqrt{1 - c^2/c_{S0}^2}}{G_1 \sqrt{c^2/c_{S1}^2 - 1}} \quad (1)$$

where, subscripts (0 and 1) denote material parameters of the substrate and the surface layer,  $G$  and  $h$  denote shear modulus and surface layer thickness, and  $c$  and  $k$  are phase velocity and wavenumber of Love wave, respectively. Love wave exists when combination of ( $c$ ,  $k$ ) satisfies above equation.

Displacement direction of Love wave is parallel to top of the surface layer, and is perpendicular to propagation direction of Love wave. Love wave is the wave which has only shear horizontal components, and it is indicated that Love wave has the same properties of the shear-horizontal SAW.

### 2.2 Viscoelasticity

In general, behaviour of solids is determined by their elasticity, and that of fluids is determined by their viscosity. In the meanwhile, behaviour of viscoelastic materials is determined by their elasticity and viscosity. The elas-

ticity and the viscosity of viscoelastic materials can be expressed as a spring and a dashpot, and are called as storage modulus and loss modulus, respectively. The viscoelastic models when the spring and the dashpot are connected in series or in parallel are known as Maxwell model or Kelvin-Voigt model, respectively. They are described as liquid-like or solid-like viscoelastic materials. In the following section, the Kelvin-Voigt model is applied to the surface layer shown in Fig. 1(b).

### 3. Love-type wave propagation simulation in viscoelastic surface layer on elastic substrate

Love-type wave phase velocity was calculated by changing storage modulus of the surface layer in harmonic analysis using finite element method.

Figure 1(b) shows a simulation model for Love-type wave propagation with a viscoelastic surface layer. The substrate has absorption layers with structural damping. Left top of the substrate was excited by an interdigital transducer (IDT), and responses by changing excitation frequency were calculated. Material parameters used in simulation were;  $G_0 = 26.32 \times 10^9$  Pa,  $\rho_0 = 2,700$  kg/m<sup>3</sup> and  $\rho_1 = 1,190$  kg/m<sup>3</sup>. The IDT parameters; electrode periodicity was 1.5 mm, aperture was 4 mm and number of electrode pairs was 3, respectively. In viscoelastic parameters, the storage modulus  $G'$  and the loss modulus  $G''$  were defined as,

$$G' = G_1, \quad G'' = \omega\eta = 2\pi f\eta, \quad (2)$$

where  $f$  is an excitation frequency of an IDT, and  $\eta$  is a viscosity of the surface layer. In this simulation, a value of  $G'$  was changed in the range of  $1.185 \times 10^6$ – $1.185 \times 10^9$  (Pa), and  $f$  was also changed from 20–1,500 (kHz) at  $\eta = 1$  Pa s.

At first, we confirmed that whether Love-type wave could propagate along the viscoelastic surface layer by changing the storage modulus. **Figure 2** shows a relationship between displacement of  $z$ -axis at top of the surface layer (measurement line in Fig. 1(b)) and storage modulus of the layer. Circles at each storage modulus denote the root-mean-square displacement exciting the IDT each frequency. From Fig. 2, it was confirmed that the displacement decreased with decreasing the storage modulus. That was suggested that Love-type wave could not propagate along the surface layer.

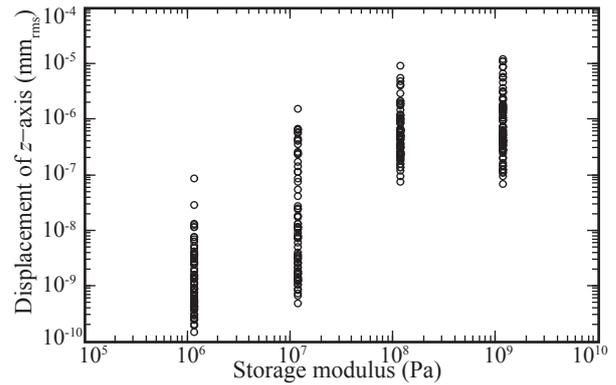
Next, we calculated dispersion curves by changing the storage modulus. **Figure 3** shows dispersion curves of Love-type wave. The dispersion curves were obtained by calculating wavenumber spectra, Fourier transform of the displacement, and converting the wavenumber domain to phase velocity domain in each frequency. Phase velocity of Love-type wave at high frequency in black part of Fig. 3(a) approached approx. 1,000 m/s, whereas the phase velocity in black part of Fig. 3(b) approached approx. 300 m/s because of decreasing the storage modulus. It was confirmed that the dispersion curves were changed by decreasing the storage modulus of the surface layer.

### 4. Conclusions

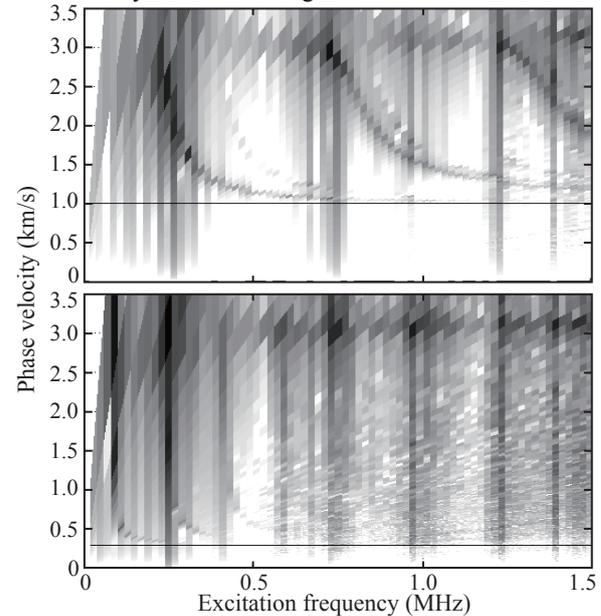
To achieve viscoelastic evaluation without contacting with piezoelectric substrate, we analyse propagation characteristics of Love-type wave in simulation, as pilot study. In this paper, displacement at top of the viscoelastic surface layer and dispersion curves of Love-type wave phase velocity were calculated with changing the storage modulus of the surface layer. It was confirmed that the displacement decrease at less than the storage modulus of  $10^8$  Pa order and the dispersion curves were changed. In future work, we will conduct that whether the theoretical and experimental dispersion curves are confirmed by using curve fitting to estimate storage and loss moduli of the viscoelastic surface layer.

### References

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**Fig. 2** Relationship between displacement of  $z$ -axis at top of the surface layer and the storage modulus.



**Fig. 3** Dispersion curves of Love-type wave phase velocity when (a)  $G' = 1.185 \times 10^9$  Pa and (b)  $G' = 1.185 \times 10^8$  Pa.