Viscoelasticity Phantom and its Quantitative Assessment Methods with Shear Wave

Mikako Gomyo1, Kengo Kondo1, Makoto Yamakawa1, and Tsuyoshi Shiina1

1. Introduction

Characterization of tissue mechanical properties, particularly elasticity, has important clinical applications because these properties are closely related to tissue condition with respect to pathology. Although numerous methods for measuring and mapping elasticity have been studied and published [1], few studies in ultrasound field have been developed to investigate shear viscosity unlike shear elasticity [2]. In many cases, Voigt model is adopted for explaining the viscoelastic properties of the material, and the parameters of the model is estimated by fitting experimental dispersion curves to the model. However, Voigt model is based on the assumption that the medium is homogenous and isotropic. In addition, the Voigt model does not always fit to the experimental dispersion curve through wide frequency range.

Here we investigated the difference of results of general rheological model analysis and Voigt model fitting by making viscoelastic phantom with various properties and using multi-line shear wave generation methods.

2. Methods and Materials

In the Voigt model, the shear wave speed \(C_S\) by frequency \(\omega\) can be fitted by (1) to inversely solve for \(\mu_1\) and \(\mu_2\).

The frequency dependent shear wave speed \(C_S\) is given by:

\[
C_S(\omega) = \sqrt{\frac{2(\mu_1^2 + \omega^2\mu_2^2)}{\rho(\mu_1 + \sqrt{\mu_1^2 + \omega^2\mu_2^2})}} \tag{1}
\]

where \(\mu_1\) and \(\mu_2\) are shear elasticity and viscosity; \(\rho\) is the density of the medium; and \(\omega\) is the angular frequency. Therefore, the value of \(C_S\) by frequency \(\omega\) can be fitted by (1) to inversely solve for \(u_1\) and \(u_2\). It seems to be reasonable, whereas, Voigt model is based on the presumption that the medium is homogenous and isotropic.

In general rheological model, the viscoelasticity is described by complex modulus \(G = G' + iG''\). The real and imaginary parts of \(G\) can be written as:

\[
G' = \rho\alpha^2 \frac{k^2 - k''^2}{(k^2 + k''^2)^2} \tag{2}
\]

\[
G'' = -2\rho\alpha^2 \frac{k'k''}{(k^2 + k''^2)^2} \tag{3}
\]

The wavenumber \(k\) is also complex, \(k = k' + ik''\) and can be calculated from a shear wave displacement \(u(x, t)\) as:

\[
u(x, t) = u_0 e^{i(\omega t - k'x)} e^{k''x} \tag{4}\]

where the \(e^{k''x}\) term accounts for the attenuation due to viscocity. Thus, assuming \(y\) is amplitude of shear wave, \(p\) is constant, \(x\) is the position in \(x\) direction, and \(q\) is coefficient of attenuation, \(k'\) and \(k''\) can be calculated as follows:

\[
k' = \frac{\omega}{C_S} \tag{5}
\]

\[
k'' = q \left( y = p \cdot e^{qx} \right) \tag{6}
\]

Therefore, if knowing both \(k'\) and \(k''\), the shear storage and loss moduli can be calculated through equations (2) and (3).

Experiments were setup for phantom study using Verasonics ultrasound systems (Verasonics Inc., Redmond, WA), each with a 128-element linear array transducer L7-4 (Philips Healthcare, Andover, MA). A frame rate is 5 kHz, and the ultrasound pushing pulse is made of 384 oscillations at a central frequency of 5 MHz, corresponding to a 76.8 us pushing time. Although data of several frequencies are needed for calculation of shear moduli, theoretically at least one pushing pulse is sufficed due to Fourier Transform. In order to improve the accuracy of viscoelasticity evaluation, we proposed multi-line shear wave generation method [3]. Thus, we make generated shear wave at different seven depths, and
this series of pushing focal line is moved laterally at twenty times at 1.5 mm intervals.

The displacements induced by shear waves at each tracking location are estimated, and these signals are converted by Fourier transform and least mean square to a phase slope $a$ per frequency $f$. $\Delta d$ is the distance between observation points. The shear wave speed at one specific place and frequency is set up as:

$$C_s(f) = \frac{2\pi \cdot f \cdot \Delta d}{a} \quad (7)$$

Viscoelastic phantom were made of gelatin and gelatin phantom. Viscosity was changed by the concentration of xanthan gum. Three kinds of homogenous phantoms were made of 8% gelatin, 10% gelatin, and mixture of 10% gelatin and 2% xanthan gum. The region of interest (ROI) is 38.4 mm wide and 27.4 mm high, and the values of shear storage and shear loss modulus are averaged inside ROI.

3. Results and Discussion

Figure 1 shows an example of measured shear storage and shear loss modulus of phantom. Shear storage modulus $G'$ increased by concentration of gelatin, while shear loss modulus $G''$ does steeply, which means xanthan gum makes medium more viscous. At several frequencies values have some variance, which is seemed to be caused by dispersion of amplitude used for calculating $k''$ involved in both shear modulus.

The multi-line shear wave generation method use many pushing pulse to improve the accuracy of viscoelasticity evaluation. Considered that condition, safety for human organ and body should be guaranteed in respect of heat. How validates this method when conducted for human is needed to be devised as well as safety. As for this experiment, these phantoms are set to be homogeneous and isotropic, so Voigt model can be thought as applicable. Assuming Voigt formula, attenuation $\alpha$ of shear wave generally can be transcribed as:

$$\alpha = \frac{\rho \omega^2 (\mu_1^2 + \omega^2 \mu_2^2 - \mu_1)}{2(\mu_1^2 + \omega^2 \mu_2^2)} \quad (8)$$

Together with (1) and (8), at a single frequency shear elasticity $\mu_1$ and shear viscosity $\mu_2$ are determined by shear wave speed $C_s$, angular frequency $\omega$, and shear wave attenuation $\alpha$ [4]. In other words, under Voigt model condition the values of shear storage and shear loss modulus can be validated by the above formula (8), the problem when applied to not-homogeneous medium like human body is remain.

4. Conclusion

This study shows that shear storage modulus $G'$ and shear loss modulus $G''$ which represent elasticity and viscosity of soft tissues, using a general rheological model, could be assessed quantitatively in different viscous medium. The results showed xanthan gum had behavior of adding viscosity to gelatin phantom. These obtained knowledge would contribute on ex vivo and in vivo experiments which are considered to handle with heterogeneous and anisotropic materials.

Acknowledgements

This research was partly supported by MEXT program for the Strategic Research Foundation at Private Universities of Doshisha University.

References


Fig. 1 An example of measured shear storage modulus ($G'$) and shear loss modulus ($G''$):