

## Generalized Harmonic Analysis of Ultrasound Waves Propagating in Cancellous Bone

一般化調和解析による海綿骨伝搬超音波の解析

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### 1. Introduction

Two longitudinal waves, called as fast and slow waves, can be observed in ultrasound signals propagating through *in vitro* cancellous bone.<sup>1</sup> The fast and slow waves have different propagation properties, which vary with a state of cancellous bone. In fact, the diagnosis of osteoporosis is performed by estimating the bone status from both propagation properties of the fast and slow waves.<sup>2</sup> However, in *in vivo* measurements, the wide overlap of the fast and slow waves are generally observed in the time domain. For more accurate diagnosis, it is necessary to clearly separate the two waves and derive the respective propagation properties. As it is difficult to separate the two waves in either of time or frequency domain, the separation methods using time-frequency analysis of short-time Fourier transform<sup>3</sup> and Wavelet transform<sup>4</sup> were attempted.

In this study, generalized harmonic analysis was applied to the ultrasound signal in cancellous bone in order to separate the fast and slow waves.

### 2. Method

The procedure of the generalized harmonic analysis is as follows.

- (1) The residual energy, which is the energy of the residual wave subtracted a sine wave from the signal for analysis, is minimized by varying the frequency and amplitude of the sine wave.
- (2) The frequency and amplitude of the sine wave minimized the residual energy is defined as those contained in the signal.
- (3) The sine wave determined in the procedures (1) and (2) is subtracted from the signal.
- (4) The residual wave obtained in the procedure (3) becomes a new signal for analysis.
- (5) The procedures (1)–(4) are repeated as required.

The sine waves obtained in the above procedures are regarded as the components contained in the signal.

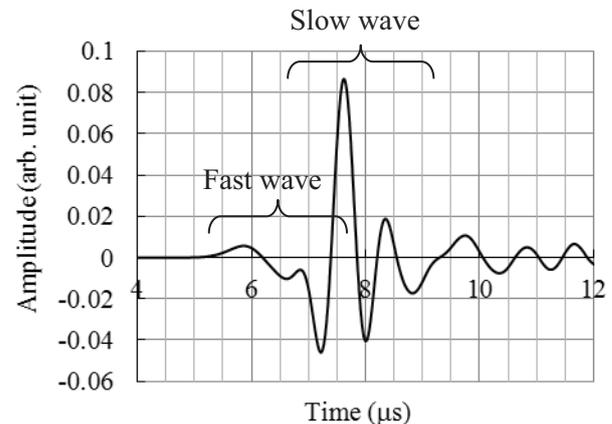


Fig. 1 Ultrasound signal in cancellous bone.

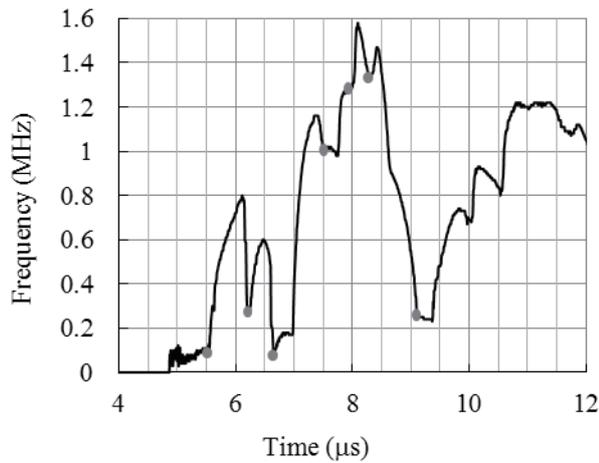
### 3. Results

Because of no random noise, the ultrasound signal obtained in the numerical simulation of ultrasound propagation in cancellous bone was used for analysis. Its waveform is shown in Fig. 1. In Fig. 1, the fast and slow waves are widely overlapped. The generalized harmonic analysis was performed for the windowed part in the time range of 0.8  $\mu$ s and the window was shifted at an interval of 8 ns. The analyzed results are shown in Fig. 2; (a) and (b) respectively show the frequency and amplitude characteristics of the components in the signal.

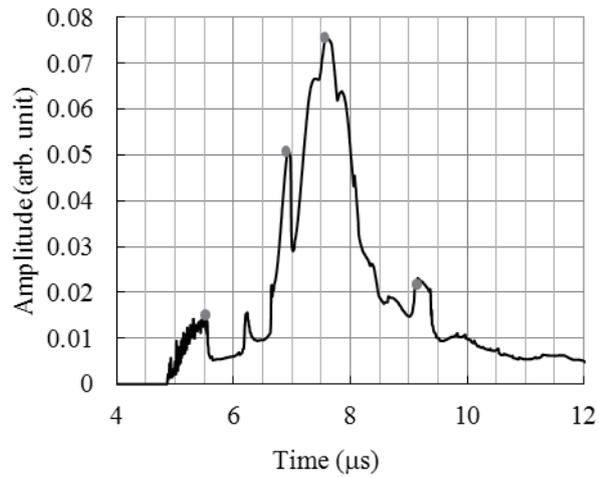
In the frequency characteristic of Fig. 2(a), several troughs can be found. For example, they appear at 5.5, 6.2, 6.7, 7.6, 7.8, 8.3, and 9.2  $\mu$ s (at the corresponding points on the graph, gray dots are drawn). Comparing with the waveform in Fig. 1, the times of 6.2, 7.8, and 8.3  $\mu$ s correspond to the zero-crossing times. The other times of 5.5, 6.7, 7.6, and 9.2  $\mu$ s doesn't correspond to the zero-crossing time but correspond to the times of the crests in the amplitude characteristic of Fig. 2(b). Moreover, at 6.7 and 9.2  $\mu$ s, the waveform in Fig. 1 is greatly changed. Therefore, it is considered that, at these times, the other wave is overlapped.

### 4. Discussion

The troughs in Fig. 2(a) appeared at the zero-crossing times or at the times at which the



(a) Frequency



(b) Amplitude

Fig. 2 Analyzed results using generalized harmonic analysis.

great change was observed in the waveform. Moreover, at the latter times, the crests appeared in Fig. 2(b). Therefore, it is considered that the start and end times of the fast and slow waves can be estimated from the trough times in Fig. 2(a) and the crest times in Fig. 2(b). Considering that the fast and slow waves are widely overlapped (this can be interpreted from the fact that the frequency characteristic greatly varied in the time range of 6–7  $\mu\text{s}$ ), it is considered that the fast and slow waves are mainly contained from 5.5 to 7.6  $\mu\text{s}$  and from 6.7 to 9.2  $\mu\text{s}$ , respectively. The center frequencies of the fast and slow waves were defined as the frequencies at which the amplitudes were largest in the respective time ranges. The amplitudes were defined as the maximum values in these time ranges.

In Fig. 2(a), the frequency at which the amplitude is largest is approximately 0.6 MHz in

Table I Fast and slow wave characteristics derived using generalized harmonic analysis.

	Fast wave	Slow wave
Time ( $\mu\text{s}$ )	5.5–7.6	6.7–9.2
Frequency (MHz)	0.6	1.6
Amplitude (Arb. unit)	0.01	0.075

the time range of the fast wave (5.5–7.6  $\mu\text{s}$ ) and approximately 1.6 MHz in the time range of the slow wave (6.7–9.2  $\mu\text{s}$ ). In Fig. 2(b), the largest amplitudes are approximately 0.01 and 0.075 in the former and latter time ranges, respectively. The characteristics of the fast and slow waves are summarized in **Table I**. Noted that the waves after 9.2  $\mu\text{s}$  were not investigated because they are neither fast nor slow wave.

The respective characteristics of the fast and slow waves were derived. However, the derived frequency and amplitude were determined from only a single component. Both the frequency and amplitude obviously varied with time as shown in Figs. 2(a) and 2(b). For more accurate estimation of the characteristics, it is necessary to investigate the variations in the frequency and amplitude with time. Moreover, although the signal with no random noise was analyzed in this study, it is necessary to investigate the effect of the random noise.

## 5. Conclusions

In this study, the separation of the fast and slow wave characteristics in the ultrasound signal was attempted by using the generalized harmonic analysis. Both the fast and slow wave characteristics could be roughly analyzed, but the variations in the frequency and the amplitude with time could not be taken into account. For more accurate estimation of the characteristics, further investigations are necessary.

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