

Output Estimation and Demonstration of Cantilever based Energy Harvesters

カンチレバー型エナジーハーベスタの出力計算手法
の提案と試作

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1. Introduction

Recently, a cantilever based micro generator using piezoelectric film is actively researched to harvest the energy from ambient vibrations and supply the energy for various autonomous applications, such as tire pressure monitoring systems (TPMS) or implantable medical devices. In these applications, it is required for the harvester to satisfy the specifications simultaneously, which are size, height, resonant frequency, output power and so on. However, a theoretical equation to design the harvester was not established well. Especially, in the micro energy harvester, deriving the solution from the strain in the piezoelectric film is extremely difficult since amplitude of vibration is too large to apply Bernoulli-Euler hypothesis. Therefore we had to spend much time for numerical calculations to design the harvester.

A new method was proposed to estimate the output power from cantilever based piezoelectric energy harvesters in this study. Simple equation to estimate the output power was derived by applying the correction coefficient which is a function of

flexure rigidity to a conventional equation. The derived equation was applied to design a practical (K,Na)NbO₃ (KNN) based energy harvester. Then, measured output from the harvester was in good agreement with the calculated value.

2. Derivation of equation

Output power from a piezoelectric energy harvester can be calculated with a following equation when amplitude of the flexure mode is small enough [2]:

$$P = \frac{\omega}{32} \frac{bh^2}{L} \frac{e_{31}^2 d}{\epsilon_{33}} \left(\frac{\partial^2 F(x)}{\partial x^2} \Big|_{x=L} \right)^2 \quad (1)$$

where, ω is a angular velocity, b , h and L are width, thickness and length of the beam, respectively, and e_{31} , ϵ_{33} and d are piezoelectric constant, dielectric constant and thickness of the piezoelectric film, respectively. Also $F(x)$ is a mode function as below:

$$F(x) = C \left[K \left(\cosh \frac{\lambda}{L} x - \cos \frac{\lambda}{L} x \right) + \left(\sinh \frac{\lambda}{L} x - \sin \frac{\lambda}{L} x \right) \right] \quad (2).$$

K and λ are constant and -1.362 and 1.875 in the fundamental mode, respectively. C is a constant determined by boundary conditions. In Eqs.(1), effect of e_{33} was ignored by Bernoulli-Euler hypothesis.

Figure 1 plots the relationship between the C and the flexure rigidity EI (E : Young's modulus, I : moment of inertia of areas) in other researchers works [3-8] using Eqn.(1) and (2). From this figure,

it was pointed out that the C is extremely high in the micro cantilever and has a linear relation for the EI though the C is invalid under the Bernoulli-Euler hypothesis. This is caused by that the effect of e_{33} cannot be ignored in the micro cantilever due to the large amplitude. In this study, instead of taking account of e_{33} , output power was estimated using linear relation between the C and EI as shown in Fig. 1. Thus, Eqn. (1) becomes

$$P = \frac{\omega}{32} \frac{bh^2}{L} \frac{e_{31}^2 d}{\epsilon_{33}} \{m_{eq} a L^3 A (EI)^B\}^2 \left[K \left(\frac{\lambda}{L} \right) (\sinh \lambda + \sin \lambda) + \left(\frac{\lambda}{L} \right) (\cosh \lambda - \cos \lambda) \right]^2 \quad (3),$$

where m_{eq} , a , and L are equivalent mass, applied

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acceleration, and length of cantilever, respectively. A and B are approximation constants derived from Fig. 1.

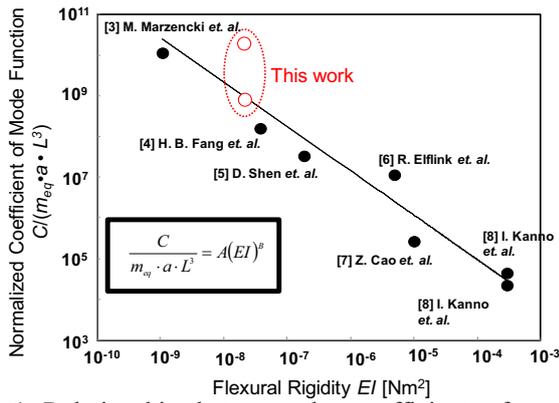


Fig.1 Relationship between the coefficient of mode function C and the flexural rigidity EI

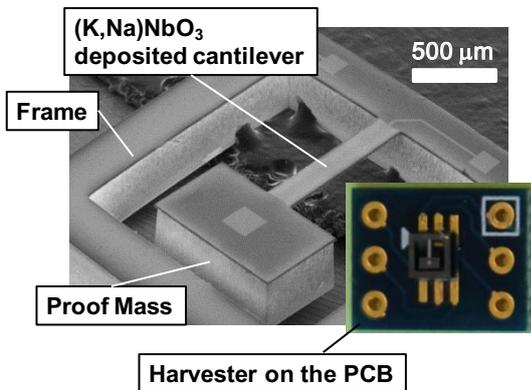


Fig.2 Relationship between the coefficient of mode function C and the flexural rigidity EI

3. Verification

To verify the equation indicated in Eqn (3), KNN based energy harvester was fabricated as shown in Fig.2 [9]. Length, width and thickness of the cantilever were 1 mm, 210 μm and 28 μm , respectively. Size of the proof mass were 600 μm \times 1 mm \times 500 μm . Also thickness of KNN film was 2 μm .

As experimental results, output power was 0.73 μW and 0.043 μW when the vibration acceleration was 9.8 m/s^2 and 0.98 m/s^2 , respectively. Resonant frequency of our device was 1.5 kHz. Figure 3 shows a performance comparison based on the normalized power density [9]. Excellent performance of our device was confirmed from Fig.3. This is caused by high figure of merit ($\text{FOM} = e_{31}^2 / \epsilon_0 \epsilon$) of KNN.

Open circles in Fig. 1 were the C 's calculated from Eqs.(1) and (2) in the fabricated device. It was confirmed that the circles were plotted in the vicinity of the expected solid line.

4. Conclusion

We proposed a new method to estimate the output power from piezoelectric energy harvesters using the flexure mode of cantilever. In the energy

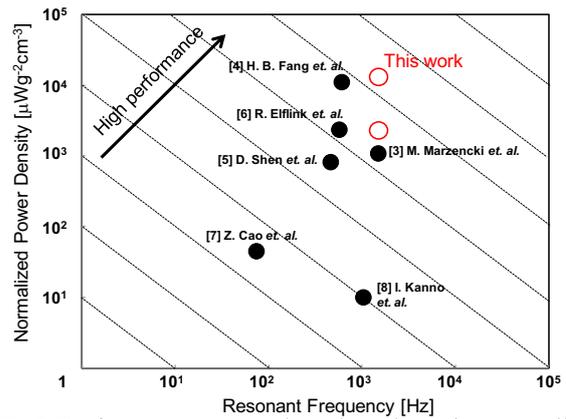


Fig.3 Performance comparison based on the normalized power density

harvester, amplitude of vibration is too large for Bernoulli-Euler hypothesis to hold true. Hence, it is difficult to derive a theoretical solution of output power. In this study, applying the correction coefficient which is a function of flexure rigidity to a conventional theoretical solution, simple equation to estimate the output power was derived. This equation was applied to design a practical (K,Na)NbO₃ based energy harvester. Then, measured output from the harvester was in good agreement with the calculated value.

Acknowledgment

We are grateful to Dr. Fumimasa Horikiri, Dr. Kenji Shibata, and Dr. Tomoyoshi Mishima at Hitachi Metal, Ltd. for their excellent advice and support. A part of this work was performed at micro/nano- machining research and education center, Tohoku University, Japan. This work was conducted under the project “Research of a nano-energy system creation” (No.18GS0203), funded by the Ministry of Education, Culture, Sports, Science and Technology (MEXT).

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