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Analysis of Quality Factor of Quartz-Crystal Tuning Forks Using L-Shaped Bar Model with Torsion Spring

ねじりバネを含んだ L 字型棒モデルによる音叉型水晶振動子の Q 値の解析

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Abstract- We analyzed the quality factor of a quartz-crystal tuning-fork using L-shaped bar model which consisted of two bars depicting the right half of the tuning fork and a torsion spring at the joint of its arm and base bars. As a result of analyzing the quality factor, it has turned out that the calculated quality factor falls down about 10% of the calculated one by a cantilever model.

1. Introduction

A formula for the dynamic capacitance of the quartz-crystal tuning fork at resonance was derived using L-shaped bar model.[1)] As a results of calculation, it has turned out that the calculated vibration efficiency of the tuning fork are slightly lower than that by a cantilever model.

Zener[2)] had elucidated the mechanism of damping in a cantilever elastic beam using thermoelasticity. Afterwards, there were some works which made Zener's analysis more precisely.[3,4)] The calculated damping coefficients had been compared with the measured ones,[3)] but the quality factor (Q value or Q) had not.

Since R reduces the efficiency of vibration a little as mentioned above, it seems that R also affects Q value of the tuning fork a little.

In this study, a formula for Q value of the quartz-crystal tuning fork has been derived by L-shaped bar model in order to predict Q value more precisely than that by a cantilever model, and a comparison between the calculated and measured Q values have been conducted.

2. Analysis

Figure 1 (a) shows the configuration of the quartz crystal tuning fork in which two arms vibrate each other in opposite directions. Figure 1(b) shows the right half of the tuning fork as an analytical model. In more detail, figure 1(c) shows L-shaped bar model which consists of a torsion spring at the joint of the base (beam A) and the arm (beam B).

The equation of motion for beam A is obtained, from the assumption on an effect of thermoelasticity not yielding in beam A as

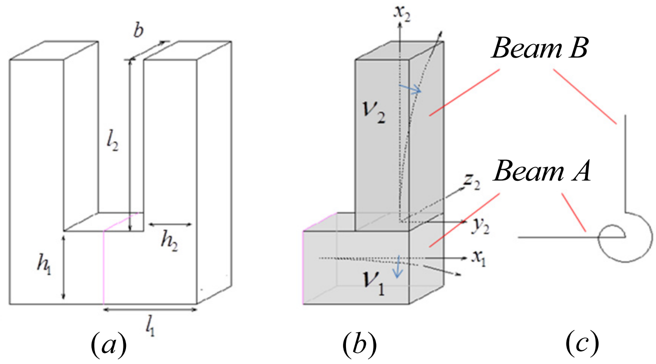

Fig.1 A quartz-crystal tuning fork and analytical models:
(a) a quartz-crystal tuning fork, (b) L-shaped bar model, and (c) a torsion spring at the joint of beams A and B.

$$E_1 I_1 \frac{\partial^4 v_1}{\partial x_1^{\ 4}} + \rho b h_1 \frac{\partial^2 v_1}{\partial t^2} = 0. \qquad (1a)$$

The equation of motion,[5)] including an effect of thermoelasticity, for beam B is given by

$$E_2 I_2 \frac{\partial^4 v_2}{\partial x_2^{\ 4}} + \rho b h_2 \frac{\partial^2 v_2}{\partial t^2} + E_2 \alpha b \int_{-h/2}^{h/2} y_2 \frac{\partial^2 \theta}{\partial x_2^{\ 2}} dy_2 = 0. \qquad (1b)$$

On the other hand, the equation of heat conduction, [3)] including an effect of thermoelasticity, for beam B when beam B undergoes flexural vibration is shown as

$$k\left(\frac{\partial^2 \theta}{\partial x_2^{\ 2}} + \frac{\partial^2 \theta}{\partial y_2^{\ 2}}\right) - \rho c_E \frac{\partial \theta}{\partial t} + E_2 \alpha T_0 y_2 \frac{\partial^3 v_2}{\partial x_2^{\ 2} \partial t} - \frac{E_2 \alpha^2 T_0}{h_2} \int_{-h/2}^{h/2} \frac{\partial \theta}{\partial t} dy_2 = 0. \qquad (2)$$

The boundary conditions employed for L-shaped bar model, that is, for beams A and B of the quartz-crystal tuning fork can be expressed as follows: (1) because one end of beam A is a fixed end at x_1=0, there is no deflection displacement and no its slope of beam A; (2) other end of beam B is a free end at x_2=l_2 and then both the bending moment and the shear force are zero; (3) at x_1=l_1 and x_2=0, beams A and B are connected on the assumption that

Table. I Comparison between the calculated and measured Q values

Resonator size ratio h_2/l_2	Qs calculated using the cantilever model	Rotational Winkler coefficient R [N・m]	Qs calculated using the L-shaped bar model	Measured Qs
0.0654	2.81×10^4	0.413	2.58×10^4	2.41×10^4
0.0947	1.24×10^5	0.442	1.12×10^5	7.09×10^4
0.0966	1.41×10^5	1.36	1.34×10^5	1.07×10^5
0.121	3.11×10^5	2.53	2.57×10^5	1.71×10^5
0.0953	1.27×10^5	2.75	1.15×10^5	8.00×10^4

Sezawa's approximation[6] is established in the frequency range for the first mode; (4) at $x_1=l_1$ and $x_2=0$, their deflection slopes of beams A and B are equal and both the bending moments of beams and the bending one arising from a torsion spring are balanced; (5) when no heat escapes out of beam B the adiabatic approximation holds on the vibrating system. After deriving the dimensionless equations from both these boundary conditions and eqs. (1a), (1b), and (2), and by applying the dimensionless boundary conditions to the solutions of the dimensionless equations rearranged from eqs. (1a) and (1b), we can obtain the following eigenvalue equation as

$$\begin{aligned}&\chi(\cosh p_2 \sin p_2 - \sinh p_2 \cos p_2)\\&\quad -(Z-\delta)(1+\cosh p_2 \cos p_2)=0.\end{aligned} \tag{3}$$

The damping of thermoelasticity can be defined by the reciprocal of Q value [4] as

$$Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right|. \tag{4}$$

Substituting the solution of the dimensionless equation of heat conduction obtained using the adiabatic boundary conditions into the dimensionless equation rearranged from eq. (1b), we have

$$\lambda^2 + p_2{}^4\left\{1+\beta\left(1-\frac{12}{\lambda\tau}+\frac{24}{(\lambda\tau)^{3/2}}\tanh\frac{\sqrt{\lambda\tau}}{2}\right)\right\}=0, \tag{5}$$

where λ is described using the dimensionless complex angular frequency ω as

$$\lambda = i\omega = i(\omega' + i\omega'') = -\omega'' + i\omega'. \tag{6}$$

Using eqs. (3)~(6), we can calculate Q values.

3. Discussion

Table I shows a comparison between the calculated and measured Q values of the tuning forks with different width h_2 and length l_2 of the arm. In Table I, Qs were measured being encapsulated in a stainless case under an evacuated condition at room temperature. The calculated Q values by L-shaped bar model has become 10% lower than that by a cantilever model and our calculated results have got closer to the measured ones.

The difference between the calculated and measured Q values shown in Table. I indicates the vibration leakage from the base to holder, is not taking into consideration in this study, the heat conduction in the length direction,[5] and an influence of thermal radiation from the beam surface to the external environment.[7]

4. Conclusion

We developed a new formula for Q value of a quartz-crystal tuning fork using L-shaped bar model which consisted of both the base and the arm of the tuning fork and a torsion spring at the joint of the arm and the base. It has turned out that the calculated Q value falls down about 10% of the calculated one by a cantilever model and approaches nearer to the measured Q value than the calculated one by a cantilever model because a torsion spring exists in L-shaped model and therefore the deflection displacement and the dissipating energy of the arm become larger than that by a cantilever model. A new formula for Q value considering the shape of the tuning fork has been derived for the first time.

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