

Numerical Analysis of Sound Wave Propagation by Combination Method using Amplitude Component of FDTD Solution and Phase Component of CIP Solution

FDTD 解の振幅成分と CIP 解の位相成分を用いた音波伝搬解析

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1. Introduction

Finite difference time domain (FDTD) method is a popular numerical method for sound wave propagation in time domain [1, 2]. It is however well known that the numerical dispersion error often occurs in the FDTD solution. On the other hand, the numerical dispersion error hardly occurs in the constrained interpolation profile (CIP) method [3-5], though the numerical diffusion error occurs. In this study, a new numerical method for sound wave propagation in time domain is proposed, in which the advantages of two numerical methods are combined. In the present method, the amplitude component of the FDTD method and the phase component of the CIP method are combined in frequency domain, and then it is transformed into time domain. Some numerical demonstrations are made for the two-dimensional sound field.

2. Combination method

In the present combination method, first the numerical solution by the FDTD method and that by the CIP method are calculated respectively in time domain for the same numerical model. Next, the frequency components of two solutions are respectively obtained by the fast Fourier transformation (FFT). Then, the amplitude component of the FDTD method and the phase component of the CIP method are extracted and combined in frequency domain, and then it is transformed into time domain by the inverse FFT.

3. Numerical experiments

3.1 Numerical model

Some numerical experiments are carried out for two-dimensional model whose size is $100 \times 100 \text{ m}^2$ as shown in Fig.1. The grid interval is assumed to be 24.4 mm, so the numerical domain is divided into 4096×4096 grid points. The Courant number is assumed to be 0.5, so the time step is $35.9 \mu\text{s}$. The boundary condition for the FDTD method is the 5-layered perfect matched layer, and that for the CIP method is the first order-absorbing boundary.

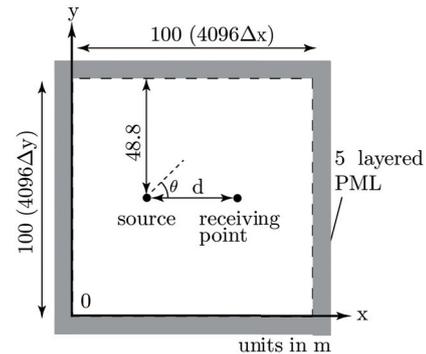


Fig. 1 Numerical model.

3.2 In the case of direct wave

A differential Gaussian pulse whose amplitude is 1 Pa and pulse width is 1.22 m is radiated from a point source located at (48.8, 48.8) m. Fig.2 shows the calculated waveform at the receiving point that is 24.4 m away from the source in the x-direction. The FDTD solution is collapsed by the numerical dispersion error. The amplitude of the CIP solution decreases by the numerical diffusion error. The solution by the present combination method shows good agreement with the theoretical solution.

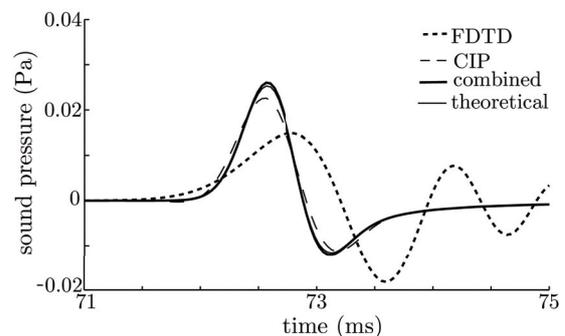


Fig.2 Sound pressure waveforms of direct wave.

Fig. 3 shows the root mean square (RMS) error of the waveform against the x -distance. The RMS error can be evaluated by the following equation

$$\varepsilon = \sqrt{\frac{1}{N} \sum_{i=1}^N [p(i) - p_t(i)]^2} \quad (1)$$

where N is number of calculation point, $p(i)$ is the

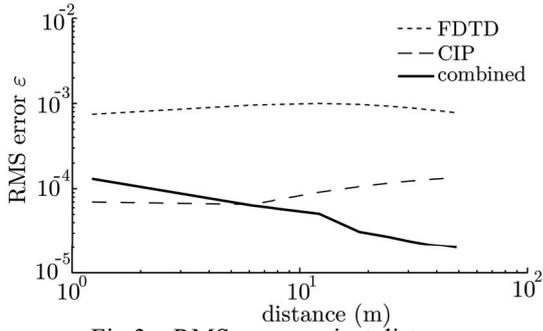


Fig.3 RMS error against distance.

calculated waveform, $p_t(i)$ is the theoretical waveform. The error of the FDTD solution becomes large in all distance because of the numerical dispersion. The error of the present combination method becomes small even if the receiving point is located far from the source, whereas the accuracy of the CIP solution degrades as the distance because of the numerical diffusion.

Fig.4 shows the performance index of each numerical scheme against the grid size of the numerical model. The performance index is evaluated as the product of the RMS error and the calculation time, so the smaller performance index indicates the faster and more accurate scheme. The present combined method shows small performance index in all grid sizes.

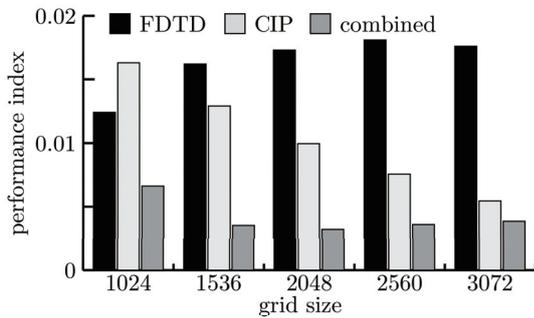


Fig.4 Performance index for various grid sizes.

3.2 In the case of reflective field

Next we discuss the case of the reflective field. Fig.5 shows the sound pressure waveform calculated by the present method when the boundary condition on the right side is assumed to be $p=0$. In this case, the distance between the source and the receiving point is 12.2 m, and that

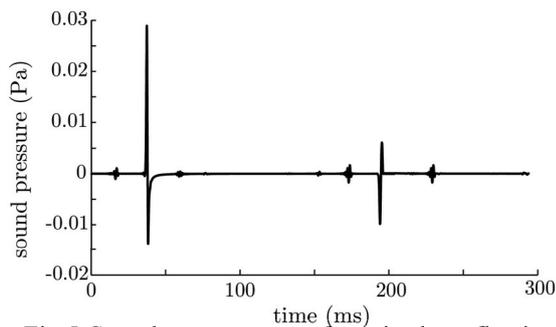


Fig.5 Sound pressure waveform in the reflective field.

between the receiving point and the reflector is 26.7 m. The reflected wave can be calculated after the

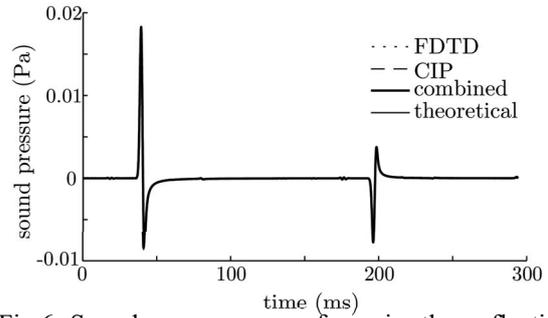


Fig.6 Sound pressure waveform in the reflective field with the longer pulse width.

direct wave, however the impulsive errors are observed. These errors are due to the periodicity of the FFT algorithm and the precision of the high frequency component.

To avoid the error, the Gaussian pulse with longer pulse width is radiated. Fig. 6 shows the sound pressure waveforms with the pulse width of 4.88 m. The direct and reflected waves are clearly

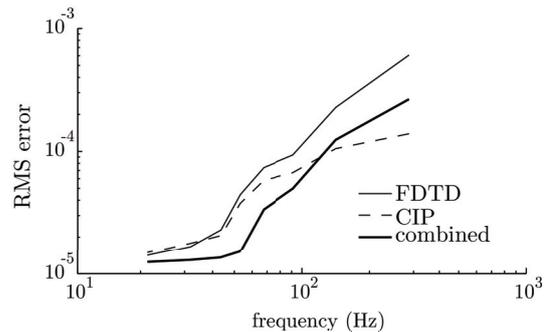


Fig.7 RMS error against frequency.

calculated without errors. It is found that the present method can be applied to the reflective sound field.

Fig.7 shows the RMS error against the frequency of the radiated pulse. In the case of higher frequency pulse, the accuracy of the present method degrades than the CIP method because of the impulsive errors. So there is suitable condition for the accurate analysis.

References

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