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A comparison of the free-surface boundary conditions in the FD-TD method for elastic wave propagation analysis
弾性波伝搬解析のための FD-TD 法における自由境界条件に関する比較

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1. Introduction

The staggered grid with the collocated grid point of velocities (SGCV) was presented for the finite-difference time-domain (FD-TD) method to model propagation of elastic waves in anisotropic solids[1)]. Since the SGCV is derived from a single control volume of the momentum conservation low and line integration of the displacement gradient, boundary conditions are simply imposed on the FD-TD method.

To demonstrate the simply imposed boundary conditions on free surfaces, a FD-TD model with SGCV was applied to resonance frequency analysis of a Lamé mode resonator on an isotropic solid, which have free surfaces between vacuum and the solid, and the results showed good agreement with the theoretical values[2)]. Although the interpolations of the velocity components are required to evaluate the velocity gradients near the free surfaces, stress components on the surfaces are simply set to zero to impose the free-surface boundary condition with the SGCV.

To compare with the SGCV, we think that FD-TD models with the conventional staggered grids (SGs) [3,4)] use more complicated procedures for the free-surface boundary condition with the stress-imaging technique[4,5)], the vacuum formalism[6)], or the adjusted staggered scheme[7)]. However, we have not discussed accuracy of results computed by SGCVs and SGs.

In this paper, resonance frequency of a Lamé mode resonator is analysed with a FD-TD method that uses a scheme of second-order accuracy in the time and spatial differences [(2,2) scheme]. Two types of grids are used, and results are compared: the one is the SGCV, and the other is a conventional SG. In the former bi-linear interpolation is used to evaluate the particle velocity on corners of unit cells, and four adjoining grids are used for interpolation to evaluate the gradients of particle velocity on grids just inside the free-surface boundaries[2)]. We have found that the accuracy of the resonance frequency with the SGCV is comparable to the one with the conventional SG with the stress-imaging technique, and that the boundary conditions can be more simply imposed on the FD-TD method with SGCV.

2. FD-TD models of a Lamé Mode Resonator

We consider a two-dimensional square Lamé mode resonator with a side length of L on an isotropic solid with Poisson's ratio 0.25[2)]. When the wavelength of the SV-wave at the frequency f_s is $2L$, the fundamental resonance frequency of the Lamé mode resonator, f_1, is $\sqrt{2}f_s$. In this paper, $f_1 = 1$ MHz and $R = v_p\Delta_t/\Delta = 0.5$, where R, v_p, Δ_t and Δ are, respectively, the Courant number, the phase velocity of the P-wave in the solid, the time interval and the grid size.

Figs. 1 and **2** show the Lamé mode resonators discretized with the SGCV and conventional SGs, respectively. In the former no grid out of the resonator is required because the free-surface boundary condition can be directly imposed as $T_{ij} = 0\ (i, j = x, y)$ for T_{ij} on the surfaces. In the latter grids out of the resonator are required for the stress-imaging technique. Here, the U and V formulations and H and V formulations[5)] are applied to the free surfaces as shown in Figs. 2 (a) and 2 (b), respectively.

3. Numerical Results

To analyze the resonance frequency of the Lamé mode resonator, the vibration and observation points are (L/4, L/4) and (-L/4, -L/4), respectively, on the x-y plane with the origin on the center of the resonator. The vibration of the x-component of the particle velocity is expressed as a sine-modulated Gaussian pulse with the center frequency f_1. The number of total time steps, N, is taken as $N = 2^{11}RL/(v_p\Delta_t)$[2)].

After the FD-TD calculation, the discrete Fourier transform is applied to the time response at the observation point in the interval from $N_s\Delta_t$ to $N_e\Delta_t$, where N_s and N_e are the numbers of time steps corresponding to the start and end sampling times, respectively, to extract the resonance

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frequency of the resonator. Here, $N_s\Delta_t = 2^8 R L v_p \approx$ 52.25 μs and $N_e\Delta_t = N\Delta_t \approx 418.05$ μs. The time response of the particle velocity at the observation point is shown in **Fig. 3**.

Table I shows the extracted resonance frequencies of the resonator. We can see that the results converge into the theoretical value [1 MHz] as L/Δ increases, and that the errors of the results with SGCV models [Fig. 1] and the ones with the SG models with U and V formulations [Fig. 2 (a)] are comparable at the same L/Δ. The result with the SG with H and V formulation [Fig. 2 (b)] for $L/\Delta = 8$ shows relatively larger error than the other results. According to Kristek *et al.*, the U and V formulations are more accurate than the H formulation[5]. The authors think that this is a cause of the larger error.

4. Conclusions

In this paper, resonance frequency analysis of a Lamé mode resonator with the FD-TD method has been carried out. The SGCV and the SG with the stress-imaging technique are used to model the resonator. In the latter models, U and V formulations, and H and V formulations are applied to the free-surface boundary of the resonator. We have found that accuracy of the resonance frequency with the SGCV is comparable to the one with the SG with the stress-imaging technique with U and V formulations, and that free-surface boundary condition can be more simply imposed on the FD-TD method with SGCV.

References

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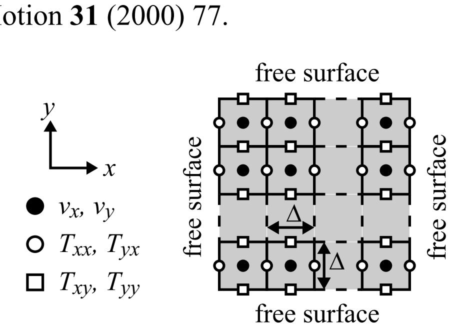

Fig. 1 A Lamé mode resonator discretized with the SGCV. The hatched region denotes the resonator.

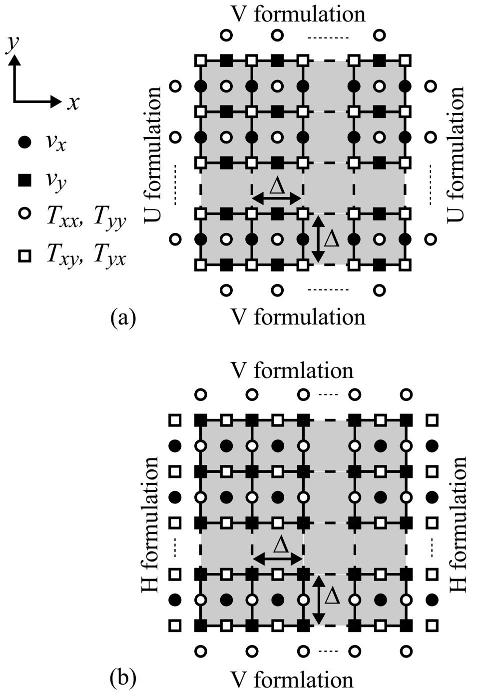

Fig. 2 A Lamé mode resonator discretized with the conventional staggered grids. Hatched regions denote the resonators. Grid points out of the resonators are for the stress-imaging technique. (a) U and V formulations and (b) H and V formulations, respectively, are used.

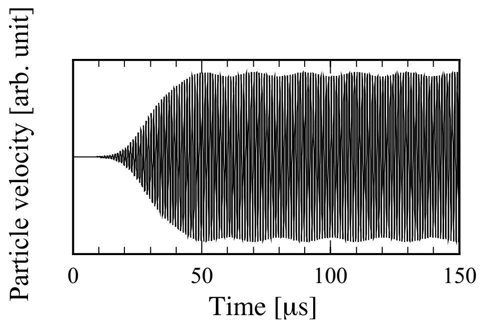

Fig. 3 Time response at the observation point for an SGCV model with $L/\Delta = 32$.

Table I Extracted resonance frequencies of the fundamental Lamé mode. Results with the SGCV and the conventional SG are shown. Here, U & V and H & V denote the results of the FD-TD model shown in Figs. 2 (a) and 2 (b), respectively.

L/Δ	Resonance Frequency [MHz]		
	SGCV	Conventional SG	
		U & V	H & V
8	1.0047	0.9951	0.9879
16	1.0023	0.9975	0.9975
32	0.9999	0.9999	0.9999
64	0.9999	0.9999	0.9999
128	0.9999	0.9999	0.9999