# Analysis and Observation of Sound Wave Field from Finite Aperture Piezoelectric Transducer - Fact-finding of Misfit between Conventional Analysis and Experiment Observation -

有限開口圧電振動子からの放射音場の解析と観測 - 従来解析が観測実験と異なる事実の解明 -

Akira Yamada1<sup>1†</sup>, and Yoshio Udagawa<sup>2</sup> (<sup>1</sup>Grad.Bio-Appl.Sys. Eng., Tokyo Univ. of A&T; <sup>2</sup>Imaging Supersonic Laboratories) 山田 晃<sup>1†</sup>, 宇田川義夫<sup>2</sup>(<sup>1</sup>東京農工大学 院生物シ応用,<sup>2</sup>アイ・エス・エル)

## 1. Introduction

Theoretical analysis of radiation sound field in a medium by finite aperture piezoelectric plate is important for the design of ultrasound transducers in nondestructive testing and/or medical imaging. Waves emitted from plate edges in addition to a primary plane wave make it difficult to understand the behavior of spatiotemporal variation of the radiated sound field.<sup>1</sup> In particular, conventional Rayleigh integral formula derived from frequency domain treatment does not agree with experiments in many aspects. There are no jaggy interferences in plane wave front amplitude. In addition, side-lobe radiation dips cannot be observed. To clear up the misunderstanding, physical behavior of the transient spatiotemporal variation of radiated sound field was investigated in detail. Simulation analysis was made using finite difference method choosing displacement as an independent variable. Mechanism of transient radiation wave field behavior from the finite aperture transducer was clarified. The results of comparative examination between simulation and photo elastic imaging experiment showed the validity of our clarification.

## 2. Strategy for sound field analysis

Finite difference time domain (FDTD) method can be used as a powerful means from solution of sound wave field by time updating numerical calculation of the difference wave equation. Validity of the method is not demonstrated especially for the case when the vibrator is excited by short period pulse signal like step function. Independent variable of the wave equation should be chosen so that transient boundary condition can be set in a appropriate way. It is noted that displacement is spatial derivative of pressure, hence, it is natural to use displacement in place of pressure filed. In addition, it matches to rectangular shaped displacement excitation assuming a single shot piston movement of the vibrator.<sup>2</sup> We therefore propose a method using displacement as an

yamada@cc.tuat.ac.jp

independent variable of the wave equation (not pressure or particle velocities as usual).

## 3. Numerical simulation

## 3.1 Method

In 2D (x,y) coordinate, piezoelectric vibrator with aperture width D is placed, center position of which is adjusted to the origin of x-axis. We consider simulation calculations for the sound wave field radiated in the semi-infinite half space y>0 when the vibrator is excited by transient pulse signal. Here, we used the 2D FDTD simulation software Wave2000® (Cyber Logic), which calculates wave field using displacement as independent variable.<sup>3</sup> A vibrator with aperture width D=12.7 mm, and center frequency  $f_0=2.25$  Mhz was considered. Material parameters were set assuming Pyrex glass with density  $\rho$ =2320 kg/m<sup>3</sup>, Lamé's constant  $\lambda$ =23879 MPa,  $\mu$ =24959 MPa, (longitudinal wave sound speed  $c_1$ =5640 m/s, shear wave sound speed  $c_s=3280$  m/s). As piston movement excitation, vibrator surface was displaced with uniform displacement at time 0. After time interval of  $T_x$  (=1/(2 $f_0$ ), it was placed back to the initial position. That is to say, time dependence of initial vibrator surface displacement is given by a rectangular function as shown in Fig.1(a). Where, for the stabilization of solution, edges of the rectangular function are rounded. Corresponding sound pressure function is as shown in Fig.1(b), which is derived from the differential calculation of displacement.



**Fig.1** Excitation waveform by piston sound vibrator: (a) displacement, (b) pressure.

#### 3.2 Displacement near vibrator edge

Simulated displacement field in the vicinity of a vibrator edge at a time  $t=0.2 \ \mu s$  is as shown in Fig.2. Where (a) is a gray scale image of displacement amplitude |u|, and (b) is a vector diagram of displacement **u**. From the vibrator surface, primary longitudinal plane wave (P wave) is emitted (red lines). In addition, rotational displacements (green lines) are generated around edge point towards the exterior direction. Cause of the rotational displacement is considered as follows. At the first half rising edge of the rectangular excitation, pushing displacement is generated toward exterior direction. On the contrary, at the latter half rising edge of the excitation, drawing displacement toward interior direction. Longitudinal edge wave called BED (Beam Edge Diffusion) wave as well as shear edge wave (Residual Shear wave) will be excited by this rotational displacement, superposing to primary plane wave field.



Fig.2 Displacement field near the edge: (a)gray scale image, (b) vector diagram (red arrow: P wave, green arrow: BED wave)



Fig.3 Simulated radiation field: (a)displacement amplitude |u| (contrast enhancement image), displacement of (b) range component  $u_{v}$ , (c)transverse component  $u_x$ , (d) pressure p.

### 3.3 Radiation sound field

Simulated radiation sound field, at a time  $t=20 \ \mu s$  when initial wave front arrived at distance y=11.3 mm is as shown in Fig.3. Where, (a) shows displacement amplitude  $|\boldsymbol{u}|$ , (b) displacement of range component  $u_{v}$ , (c) transverse component  $u_x$ , (d) pressure p. It can be

seen that BED having cylindrical wave-front emanating from both edges propagate, superposing to the primary plane wave field. We can also recognize that shear edge wave (RS) propagates at the delayed time having similar cylindrical wave-front. Note that rotation direction of RS is opposite to BED. Pressure field p, calculated from divergence (space derivation) of u, is shown in Fig.3(d).



We can see that sign of BED in displacement filed is unipolar, on the other hand, that in pressure fields is bipolar. That is, exterior BED in pressure field has positive sign, on the other hand, interior part has negative.

waves from vibrator edge.

### 4. Photoelastic visualization experiment

Observation was made using photoelastic visualization experiment apparatus. Piezoelectric circular plate with diameter  $\phi$  40 mm and center frequency  $f_0=0.5$  MHz was prepared. Step voltage was applied to the plate and emitted to a Pyrex glass cuboid block (50x50x15 [mm]). The plate was backed with same piezoelectric material, so that returning backward wave was suppressed. Since thickness of the test piece was small, radiated sound field was assumed to be 2D. Measured photoelastic image of radiated sound field is shown in Fig.5(a). Corresponding simulation image is also compared as shown in Fig.5(b). Sound propagations of plane wave, BED, and RS are clearly visible. They show very close appearance between simulation and experiment.



(a) Experiment (b) Simulation Fig.5 Visualization image, (a) experimented photoelastic image, (b)simulated image.

#### References

- 1. Y.Udagawa, Introduction to Ultrasonic Wave Technique Transducer, Pulser, Receiver and Device-, Nikkan Kogyo Shimbun, Tokyo (2010).
- 2. Y.Udagawa and A.Yamada, "Simulation and Verification Experiment of Radiation Sound Pressure Waveform from Finite Aperture Piezoelectric Transducer", The 34th Symp. Ultrason. Elect. (2013).
- 3. R.S.Schechter, H.H.Chaskelis, R.B.Mignogna, P.P.Delsanto, Science, 265 (Aug.1994) pp.1188-92.