# Phonon Propagation in Solid-liquid Superlattices

固体液体超格子におけるフォノンの伝播特性

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# 1. Introduction

The noticeable behavior of phonons in periodic superlattices (SLs) originate mainly from the existence of frequency gaps, which are due to the Bragg reflections of the phonons with long wavelengths [1, 2]. Since Bragg reflections occur essentially as a result of the interference of waves, they are also seen in the propagation of photons and electrons through periodic media [3]. In the case of phonons, there exists an inherent type of Bragg reflection in addition to ordinary Bragg reflections [4, 5].

When phonons are injected to interfaces of a SL at an angle, three modes of propagation, i.e., one longitudinal (L) and two transverse (T), are involved in the reflection and transmission processes. This leads to coupling between the modes and mode conversion at the interface. As a result of the multiple reflection by the interfaces of the periodic SL, there exist two types of Bragg reflection, i.e., "intermode" Bragg reflections can besides ordinary "intramode" occur Bragg reflections[4, 5]. For example, incident L phonons can be Bragg-reflected as T phonons, i.e., the amplitudes of the reflected T phonons add up in phase but those of the reflected L phonons are canceled out. Therefore, the corresponding frequency gaps should be classified when mode-dependent characteristics of phonon propagation are investigated [5].

Moreover, for phonons in superlattices consisting of solid and liquid layers, another inherent type of interference effect can be expected, because there is no transverse phonons within the liquid layers.

That is, when L phonons are injected to an interface of liquid and solid layers at an angle from a liquid, three modes of transmission are involved. As for reflected phonons, however, only the longitudinal mode is involved. This leads to an interesting mode-dependent characteristics of phonon propagation.

Recently, Hassouani *et al.* studied the sagital acoustic waves in finite solid-liquid SL based on the Grren's function method [6]. They pointed out the peculiar properties of solid-liquid SLs are the existence of two types of frequency gap, i.e., the stop bands originating from the periodicity of the system (Bragg-type gap) and the transmission zeros induced by the presence of the solid layers immersed in the liquid. However, the physical meaning of the transmission zeros has not been clear.

In the present study, based on a different method, we caclulate the dispersion relations, transmittance of phonons, and the corresponding velocity and stress fields, and discuss the properties of phonons in the solid-liquid super lattices.

## 2. Method

In the present calculation, we adopt the isotropic continuum approximation for solid layers of the SL. Under this condition, the phonon modes polarized in the sagittal plane are decoupled from the horizontally polarized shear (SH) mode. We consider sagittal modes, i.e., the coupled L and T vibrations in the sagittal plane.

In addition, liquid layers are assumed to be ideal. This leads to the conditions that viscous shear stresses vanish but tangential displacements need not to be continuous at the interfaces between solid and liquid layers. As a result, the normal stress and the normal velocity should be continuous at the interfaces. These conditions are formulated in terms of the transfer matrix.

Based on the transfer matrix method, we calculate the dispersion relations of solid-liquid SLs with the infinite number of unit periods. Moreover, transmittance and phase time of phonons propagating through the finite solid-liquid SLs are calculated. In the present proceedings, only the dispersion relations are shown.

## 3. Numerical results and discussions

As a numerical example, we show in Fig. 1 the dispersion relation of the SL consisting of Plexiglas and water layers as a function of the wave vector  $k_{\parallel}$  parallel to the interfaces. The gray and white areas correspond to the frequency bands and gaps, respectively. The thicknesses of the solid and liquid layers are assumed to be the same. Parameters we used are as follows:  $\rho = 1.20$  g/cm<sup>3</sup>,

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 $v_t$  = 1.38 km/s, and  $v_\ell$  = 2.70 km/s for Plexiglas;  $\rho$  = 1.00 g/cm<sup>3</sup>,  $v_\ell$  = 1.49 km/s for water [6].

Our calculations reproduced the results of Hassouani *et al.*, though the calculation method is different. The region marked with \* corresponds to transmission zeros.



**Fig. 1** Phonon dispersion relations of a superlattice constituting of Plexiglas and water as a function of the wave vector  $k_{\parallel}$  parallel to the interfaces. The gray and white areas correspond to the frequency bands and gaps, respectively. The thicknesses of the solid and liquid layers are assumed to be the same. The region marked with \* corresponds to transmission zeros.

Figure 2 shows the phonon dispersion relations calculated for  $\theta = 0^{\circ}, 20^{\circ}$ . Bloch wave number  $k_z$  and the imaginary part of  $k_z$  are shown as a function of the frequency. In the Bragg gap, the imaginary part of the Bloch wave number is a continuous function of the frequency. On the other hand, in the transmission zeros, the imaginary part of the Bloch wave number is not a continuous function and becomes infinity. This is a characteristics of the transmission zeros.

### 4. Conclusions

We calculated the dispersion relations of a superlattice consisting of alternate stacking of liquid and solid layers, and examined the vibrational modes in this structure. The phonon velocity and stress fields will be illustrated elsewhere and also the detailed characteristics will be examined. In particular, the vibrational modes within the transmission zeros will be discussed.



**Fig. 2** Phonon displacements in a superlattice constituting of Plexiglas and water for  $\theta = 0^{\circ}, 20^{\circ}$ . Bloch wave number  $k_z$  (solid lines) and the imaginary part of  $k_z$  (broken lines) are shown as a function of the frequency.

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