# Anisimkin's (Quasilongitudinal) Modes in Various Material

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## 1. Introduction

In 2004, Anisimkin, Jr., found numerically verified experimentally peculiar modes and propagating along the x (diagonal) axis in a quartz ST cut plate. They are quasilongitudinal (QL), that is dominant displacement is nearly uniform through the plate thickness with little horizontal and vertical shear displacements. Its velocity is close to the longitudinal bulk wave. They exist at nearly periodic values of  $\xi h$  (wave number along x axis by half thickness).<sup>1)</sup> multiplied Gulyaev numerically searched similar modes in LiNbO<sub>3</sub> and Te. He found, in LiNbO<sub>3</sub>, QL modes exist for small value of  $\xi$ h, but not at periodic values. In Te, he noticed "an unusually wide ranging QL modes".<sup>2)</sup> Anisimkin, Sr., conducted further numerical search of QL modes in various material, but in only two specific cut angles, namely  $(0^{\circ}, 130^{\circ}, 0^{\circ})$  and  $(89^{\circ}, 130^{\circ}, 0^{\circ})$  $37^{\circ}$ , 104°) in his notation.<sup>3)</sup>

In previous papers, we presented analyses of QL modes.  $^{4)}$   $^{-6)}$  In this paper, based on these analyses, we present a quick method for searching QL modes of waves propagating along the x-axis in a rotated Y-cut plate. The method is general enough to cover trigonal, monoclinic, orthorhombic, hexagonal, isometric and a part of tetragonal (422, 4mm, 42m and 4/m2/m2/m) crystal systems.

After a brief analytical description of the search method, we examine various material and find numerous QL modes, which have not been reported.

### 2. Analytical description of the search method

Since piezoelectricity causes no substantial effect on mode shape, the first search is done in pure elastic case. Then full piezoelectric analysis is applied to candidates obtained in the first search.

Displacements of symmetric modes are given by:

$$\begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \end{pmatrix} \propto \begin{vmatrix} \sin(\xi \mathbf{x}_1) \ast \cos(\lambda \xi \mathbf{x}_2) \\ \cos(\xi \mathbf{x}_1) \ast \sin(\lambda \xi \mathbf{x}_2) \\ \cos(\xi \mathbf{x}_1) \ast \sin(\lambda \xi \mathbf{x}_2) \end{vmatrix}$$
(1)

In order to satisfy the equation of motion, the following determinant shall be zero.

$$det(\Gamma A(\lambda)) = 0$$
(2)  
where  
$$\Gamma A(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 & \lambda \\ 0 & \lambda & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \lambda & 1 & 0 \end{pmatrix} * Mn$$
$$* \begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \\ 0 & 0 & 1 \\ \lambda & 1 & 0 \end{pmatrix} + \begin{pmatrix} -Cpn^2 & 0 & 0 \\ 0 & -Cpn^2 & 0 \\ 0 & 0 & -Cpn^2 \end{pmatrix}$$
(3)

Mn is a stiffness matrix after the rotation. Monoclinic symmetry is good enough to cover crystal systems mentioned in the Introduction.

For a given value of Cpn, phase velocity normalized by the longitudinal wave velocity, the determinant is a bi-cubic equation of  $\lambda$ . For Cpn=1, its one root becomes to zero and hence the equation reduces to a bi-quadratic equation. As explained in previous papers, QL modes appear when the remaining two roots are nearly equal. Hence behavior of the discriminant of the bi-quadratic equation is a good measure to search QL modes.

Another possibility of QL modes occurs when SH waves are decoupled with P+SV modes. There is one and only one  $\lambda$  and hence QL modes always appear at periodic values of  $\xi$ h. This is very much similar to isotropic case, of which SV component vanishes, when the phase velocity is equal to the longitudinal velocity.<sup>7)</sup>

## 3. Some results of search

Behaviors of the discriminant as a function of rotation angle of rotated Y cut of various material are calculated. Its zero or minimum yields a good candidate of QL modes. Then a full piezoelectric analysis is applied to see details of its displacements at the surfaces and across the thickness..

Only a few samples are presented here, others will be presented elsewhere.

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Fig.3 Displacements across the thickness of LiNbO<sub>3</sub> for ξh=4.76

### 3.1 LiNbO<sub>3</sub>

Fig.1 shows behavior of the discriminant for rotated Y cut of LiNbO<sub>3</sub>. A minimum close to zero is seen at 135°. A full piezoelectric analysis yields Fig.2 and Fig.3, displacements across the thickness for  $\xi$ h of 2.38 and 1.52, respectively. They are QL modes, although the longitudinal displacement is not uniform inside of the plate.

A similar behavior was also found in LiTaO<sub>3</sub>.

## 3.2 ZnO

In Y cut of ZnO, SH waves are decoupled with P+SV modes. Hence QL modes always exist at periodic values of  $\xi$ h.

A full analysis yields **Fig.4** and **Fig.5**, displacements across the thickness for  $\xi$ h of 1.63 and 3.25, respectively.

## References

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Fig.4 Displacements across the thickness of ZnO for  $\xi h = 1.63$ 

