Measuring suspension dynamics through phase correlations in Diffusing Acoustic Wave Spectroscopy (DAWS)

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1. Introduction

In weakly scattering dynamic suspensions, the motion of the scatterers can be measured by measuring the change in phase of reflected ultrasonic waves, forming the basis of the well-known technique of Doppler ultrasound. In more strongly scattering media, however, these methods break down, and the suspension dynamics must be understood through correlations and statistics of multiply scattered waves [1]. Because multiply scattered waves sample the medium many times, they are extremely sensitive to small changes, and therefore provide an excellent tool for probing the dynamics of complex systems [2].

This sensitivity has been exploited using both optical and acoustic waves in the fields of Diffusing Wave Spectroscopy (DWS) and Diffusing Acoustic Wave Spectroscopy (DAWS) with much success. However, because it can be difficult to measure directly (in the case of optical waves), and because of its seemingly random nature, the phase of the waves is often ignored. Here, we measure temporal fluctuations of the phase of multiply scattered acoustic waves transmitted through dynamic media. This phase information yields new information about the motion of the scatterers, going beyond traditional DAWS measurements.

2. Method

In DAWS, a short ultrasonic pulse is incident on the medium, and the transmitted field is recorded. A short time later, after the scatterers have moved a small distance, the pulse is repeated, and the transmitted field is now slightly different – each pulse essentially acts as an ultrasonic "snapshot" of the system (c.f. **Fig. 1**).

The decorrelation time of the field amplitudes τ_{DAWS} is the characteristic time of the system. By measuring the field correlation function $g_1(\tau)$, the mean square displacement $\langle \Delta r_{rel}^2(\tau) \rangle$ of the scatterers can be found as a function of time, up to several τ_{DAWS} . Similar information can be obtained by considering the phase statistics and correlations of the transmitted field; however, the phase yields a more robust and sensitive measurement of this

quantity [3]. Furthermore, by *unwrapping* the phase, so that it is no longer constrained to $\pm \pi$, the *cumulative* phase correlation function [4] can be measured, yielding new and much longer time-scale information about the system.



Fig 1 Wave transport in DAWS. The particle positions \mathbf{r}_p have moved a distance $\Delta r_p(\tau)$ in the time interval τ , resulting in a modified scattering path length, altering the phase of the wave.

2. Experiment

Two types of scatterers were studied in these experiments: glass spheres and bubbles. The 1-mm-diameter glass spheres were contained in a 12.2-mm-thick fluidized bed, suspended at a volume fraction of 40% by an upward flowing solution of 60% glycerol and 40% water. The bubbles were produced through electrolysis in a 5.5-mm-thick cell, designed to create turbulent flow for bubbles in the 10–100 µm size range. А miniature hydrophone was used to capture the field transmitted through the sample in a single near-field speckle spot. The input pulses had a central frequency of 2.25 MHz, were only a few periods wide, and were repeated every 2 ms. At a fixed time lapse after each input pulse, a short segment of the transmitted waveform was recorded. The phase and amplitude of the wave as a function of time was determined using a numerical technique equivalent to a Hilbert transform.

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3. Results and Discussion

The mean square displacement of the glass beads was measured using the phase probability distribution and variance, and compared with the traditional DAWS correlation approach. The results from all three methods are in very good agreement, and yield a τ_{DAWS} of 89 ms. Also, the measurements from the phase variance are seen to be more robust for small τ . This is exemplified in the inset of **Fig. 2**, which shows the effect of 2% gain fluctuations in the data – the field correlation data is degraded while the phase measurement is unaffected.



Fig 2 The relative mean square displacement of the glass beads determined from the phase distribution and variance, compared with traditional DAWS measurements. The inset shows the effect of amplitude noise (see text).

Further information about the scatterer dynamics can be obtained through the phase derivative distributions, shown in **Fig. 3**. These distributions provide a sensitive probe of the early time behaviour of the particle motion, in powers of $x = \tau/\tau_{DAWS}$: $\langle \Delta r_{rel}^2 = 324x^2 - 57x^4 - 3.3x^6 \rangle$. Not only do the data fit the theoretical predictions extremely well, these data provide details about the data up to the 6th power in time, which would be impossible from conventional DAWS methods.



Fig 3 Phase derivative distribution functions, measured from glass bead experiments.

In the bubble experiments, the unwrapped phase was used to gain additional insight into the long time-scale dynamics of the system. **Fig. 4** shows the cumulative phase correlation function, which shows information about the scatterers up to $25\tau_{DAWS}$ – approximately 3 times longer than measureable by the field correlations or wrapped phase! In particular, a fit to these data show a crossover from ballistic to diffusive bubble dynamics at around $5\tau_{DAWS}$, and an average velocity parallel to a concentration gradient of the bubbles.



Fig 4 Cumulative phase correlation function, as measured from bubble experiments.

4. Conclusions

We have investigated the dynamics of two types of suspensions through measurements of the phase of transmitted, multiply scattered ultrasonic waves. The excellent agreement of theory and experiment has allowed us to relate the observed fluctuations in phase evolution to the relative mean square displacement of the scatterers. The phase statistics and correlations are sensitive probes of the particle motion on both short and long time scales, and can provide more accurate information than the more traditional field fluctuation measurements.

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References

- M.L. Cowan, J.H. Page, D.A. Weitz: Phys. Rev. Lett. 85 (2000) 453; M.L. Cowan *et al.*: Phys. Rev. E 65 (2002) 066605.
- R. Snieder and J.H. Page: Phys. Today 60 (2007) 49.
- 3. M.L. Cowan *et al.*: Phys. Rev. Lett. **99** (2007) 094301.
- 4. B.A. van Tiggelen, D. Anache and A. Ghysels: Europhys. Lett. **74** (2006) 999.