

Issue in adoption of lumped-parameter circuit for electromechanical coupling phenomenon and its improvement --- Modifying circuit structure

電気機械結合集中定数回路の問題点と改善策 ～回路構成の修正～

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1. Introduction

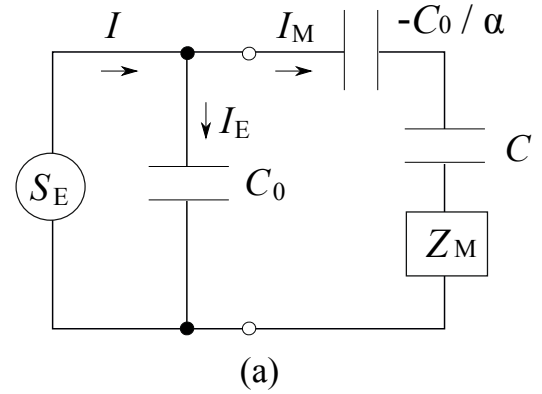
The characteristics of electromechanical coupling transducer can be represented using lumped parameters, in which a pair of positive and negative capacitance of the dielectric component, $\pm C_0$, appears. However, some problems occur in the conventional framework of the circuit, as discussed later. Although a fundamental solution is to discard the lumped-parameter elements,¹⁾ in this study, another solution is proposed within the framework of lumped-parameter circuit by modifying the circuit structure.

2. Summary of conventional equivalent circuit and its defects

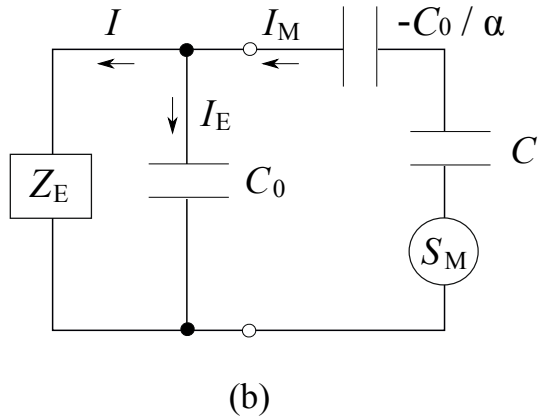
Figure 1 shows the essential framework of the conventional equivalent circuit for the electromechanical transducer at the low frequency limit, $\omega \rightarrow 0$. In Fig. 1 (a), S_E and Z_M indicate the electrical source and the mechanical load, respectively, while in Fig. 1 (b), S_M and Z_E indicate the mechanical source and the electrical load, respectively. C is the elastic component, and all the vibration modes n contribute to C , as in $C = \sum_n C_n$, at $\omega \rightarrow 0$. With regard to $-C_0/\alpha$, $\alpha = 1$ in the piezoelectric longitudinal (L-) effect, while $\alpha \rightarrow 0$ (shorted out) in the piezoelectric transverse (T-) effect.

For the later discussion, the energy conversion efficiency and the resonance frequency shift in the electromechanical coupling is summarized.

The electromechanical coupling coefficient k^2 is calculated with the ratio of the stored elastic energy to the input electrical energy at the situation of impedance $Z_M \rightarrow 0$ (no-load) in Fig. 1 (a), or the ratio of the stored dielectrical energy to the input mechanical energy at the situation of impedance $Z_E \rightarrow \infty$ (no-load) in Fig. 1 (b). The former is given



(a)



(b)

Fig. 1: Conventional equivalent circuit. ($\omega \rightarrow 0$)

by $k^2 = C_{\text{eff}}/(C_{\text{eff}} + C_0)$ with $C_{\text{eff}} = 1/(-\alpha/C_0 + 1/C)$, which leads to

$$k^2 = C/C_0 \quad \text{for L-effect} \quad (1)$$

$$k^2/(1 - k^2) = C/C_0 \quad \text{for T-effect} \quad (2)$$

The latter is given by $k^2 = (1/C_0)/(1/C_{\text{eff}} + 1/C_0)$, leading to the same result as the former.

The acoustic velocity is affected by the impedance of the electrical source or load. When the load impedance $Z_E \rightarrow 0$ in Fig. 1 (b), the inverse of the total capacitance of the resonance system, $1/C_{\text{total}}$, is estimated as

(i) $1/C_{\text{total}} = 1/C - 1/C_0$ for L-effect,

(ii) $1/C_{\text{total}} = 1/C$ for T-effect,

while when $Z_E \rightarrow \infty$,

(iii) $1/C_{\text{total}} = 1/C$ for L-effect,

(iv) $1/C_{\text{total}} = 1/C + 1/C_0$ for T-effect.

The state (i) is the electromechanical coupled state for L-effect, while the state (iii) is the “intrinsic” state for L-effect. The acoustic velocity at state (i) is smaller than at state (iii), each corresponding to a resonance and an antiresonance, respectively, observed electrically in the circuit in Fig. 1 (a). On the other hand, the state (iv) is the electromechanical coupled state for T-effect, while the state (ii) is the intrinsic state for T-effect. The acoustic velocity at state (iv) is larger than at state (ii), each corresponding to the electrical antiresonance and resonance, respectively.

The conventional equivalent circuit has the following disadvantages:

(I) The admittance observed in the circuit of Fig. 1 (a) is reduced near the electrical antiresonance, and skews. However, the mechanical vibration level actually does not indicate such a dip at all near the antiresonance, and neither does the input power level. Therefore, the inverse Fourier transform of the skewed admittance does not provide the correct impulse response of the system.

(II) Since $k^2 < 1$ and $C_{\text{total}} > 0$ are required, the value of C/C_0 is restricted to $C/C_0 < 1$ in the case of L-effect, while no such restriction is imposed on the case of T-effect. This unnatural asymmetry suggests that the circuit structure shown in Fig. 1 for L-effect becomes incorrect as $C/C_0 \rightarrow 1$.

3. Modification to equivalent circuit

For the purpose of removing the defects mentioned above, the modified circuit model shown in Fig. 2 is proposed. The circuit is composed of two parts (circuits A and B) connected through an energy transfer element labeled TR in the figure. The function of source is considered in two steps. The energy from the 1st source is transferred via TR into the 2nd source. In Fig. 2 (a), the voltage at C_0 on the circuit A is transferred to C_0 on the circuit B which functions as a voltage source with “internal capacitance” C_0 . In Fig. 2 (b), the charge at C_0 on the circuit B is transferred to C_0 on the circuit A which functions as

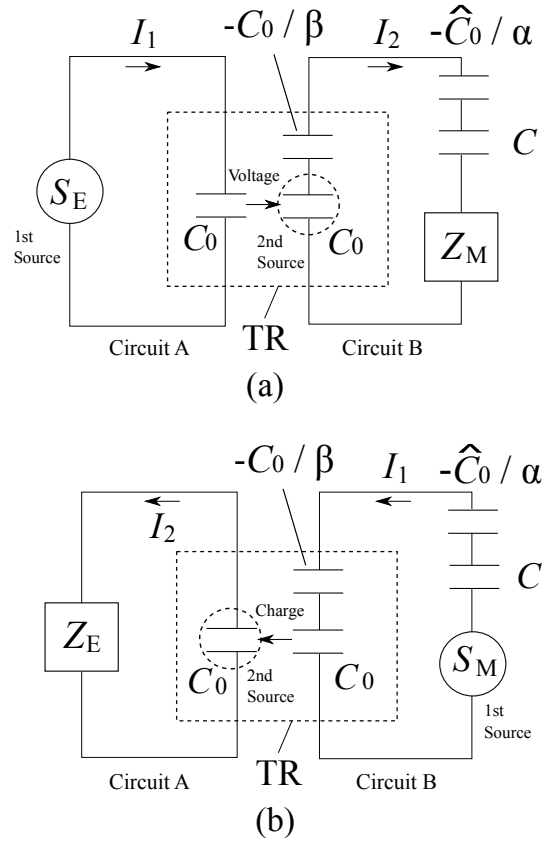


Fig. 2: Modified equivalent circuit.

a charge source (similar to the current source) with internal capacitance C_0 . Another negative capacitance $-C_0/\beta$ is newly introduced, which reflects the impedance of electrical source or load:

$$\beta \rightarrow 0 \quad \text{for electrical open state,} \quad (3)$$

$$\beta \rightarrow 1 \quad \text{for electrical short state.} \quad (4)$$

By introducing the two-step sources, the mechanical vibration and input power level can be grasped properly.

Moreover, $-C_0/\alpha$ in Fig. 1 is replaced with $-\hat{C}_0/\alpha$ in Fig. 2, where

$$-1/\hat{C}_0 = -1/C_0 + 1/C_\Delta, \quad (5)$$

$$1/C_\Delta \rightarrow 0 \quad \text{for } C/C_0 \rightarrow 0. \quad (6)$$

Then, k^2 is recalculated as

$$k^2 = \frac{C}{C_0} \times \frac{C_\Delta}{C + C_\Delta}, \quad (7)$$

which lifts the restriction of $C < C_0$.

1) M. Ohki: Jpn. J. Appl. Phys. 53, 07KB03 (2014).