Piezoelectric nonlinear vibration focusing on the second harmonic vibration mode

Ryohei Ozaki1†, Yaoyang Liu1 and Takeshi Morita1
(1 Graduate School of Frontier Sciences, The Univ. of Tokyo)

1. Introduction
The piezoelectric materials are used in resonant type devices which are driven under high power condition. Usually, these piezoelectric devices are designed using FEM. In FEM calculation, only the linear piezoelectric effect is taken into consideration. However when the piezoelectric devices are driven under high power condition, the nonlinear piezoelectric vibration becomes apparent. In that case, the simulated results differ from the actual one. To overcome this problem, the FEM involving nonlinear piezoelectric vibration should be developed. The purpose of this study is to establish the nonlinear model of the piezoelectric vibration for FEM. Differently from the previous studies focusing on the third term nonlinearity1,2, we studied about the second term nonlinearity because it is larger than that of third term coefficient.

2. Nonlinear model
In this study, the plate type piezoelectric transducer was utilized (Hard type, C203 Fuji ceramics). The material constants of this transducer are shown in Table 1. In this model, x origin is the center of the transducer and we measured the longitudinal vibration. The heat generation also affects the nonlinearity3; therefore to eliminate the effect of heat generation, we established the measurement system using function generator (WF1948), laser doppler velocimeter (NLV2500), high speed amplifier (HSA4052) and lock-in amplifier (LI5640). In this system, the interval time between vibration excitation can be controlled. With this control, the temperature was kept constant.

In the linear model, the stress and the strain are expressed as proportional,

\[ S_1 \left( \frac{\partial u}{\partial x} \right) = s_{11}^E T_1. \] (1)

where \( S_1, T_1, s_1 \) and \( u \) are the strain, the stress, the compliance and the vibration displacement, respectively.

We added the second term \( s_{11}^E(2)T^2 \) to explain the effect of the second harmonic vibration mode as

\[ S_1 = s_{11}^E T_1 + s_{11}^E(2)T_1^2. \] (2)

By integrating this equation, we obtain the vibration displacements as a function of \( x \),

\[ u(x) = \int_0^x s_{11}^E T_1 dx + \int_0^x s_{11}^E(2)T_1^2 dx. \] (3)

In this equation, the part of second harmonic \( u_2 \) is defined as

\[ u_2(x) = \int_0^x s_{11}^E(2)T_1^2 dx. \] (4)

The stress \( T_1 \) was adopted based on the linear model.

\[ T_1(x) = T_{a1} \cos \left( \frac{\omega x}{c} \right) \cos(\omega t) \] (5)

where \( \omega_r \) is resonant frequency and \( c \) is sound velocity, respectively.

Finally the second harmonic frequency on each position \( x \) is deduced as

\[ u_2(x, t) = s_{11}^E(2)T_{a1} \left( \frac{\omega x}{c} + \frac{L}{4\pi} \sin \left( \frac{2\pi x}{L} \right) \right) \cos(2\omega t) \] (6)

where \( L \) is transducer length.

By measuring the displacement of second harmonic vibration, we can calculate the nonlinear...
3. Mode shape of the second harmonic vibration

To obtain the nonlinear parameter, we measured the mode shape of second harmonic vibration at the resonant frequency (35.5kHz) with 100Vpp input voltage. This transducer is symmetric so we measured the half of the transducer. Figure 2 shows the mode shape of the transducer and the fitting curve with our model.

The fitting result was well fitted and the value was calculated to be $3.6 \times 10^{-21} \ [m^4/N^2]$ from this measurement.

4. The $s_{11}^{E(2)}$ calculation from the velocity at the tip of the transducer

The amplitude of the stress at the center of transducer ($T_{a1}$) and that of velocity with driving frequency at the tip ($v_{a1}$) are also proportional in linear model expressed as

$$T_{a1} = \rho c v_{a1}. \quad (7)$$

From the equation (6), the displacement of the second harmonic vibration at the tip of transducer is

$$u_2(L,t) = s_{11}^{E(2)} T_{a1} L \sin(2\omega t). \quad (8)$$

By calculating the temporal differentiation of equation (8), the velocity is expressed as

$$v_2 \left( \frac{L}{2}, t \right) = \frac{\partial u_2(L,t)}{\partial t} = s_{11}^{E(2)} T_{a1} L \omega \sin(2\omega t) = v_{a2} \sin(2\omega t) \quad (9)$$

From equation (7) and (9), the relationship between $v_{a2}$ and $v_{a1}$ is expressed as

$$\frac{v_{a2}}{\omega} = s_{11}^{E(2)} \frac{v_{a1}^2}{2} \quad (10)$$

Using a lock-in amplifier (LI5640), we measured the velocity with the driving frequency and that with the second harmonic frequency. The measurement result and the fitting curve with equation (10) were shown in Fig. 3. From this curve fitting, the calculated value of $s_{11}^{E(2)}$ was calculated to be $3.4 \times 10^{-21} \ [m^4/N^2]$. This value was the almost same to the value calculated from mode shape measurement ($3.6 \times 10^{-21} \ [m^4/N^2]$). This curve fitting indicates that the value of $s_{11}^{E(2)}$ can be treated as a constant parameter regardless of frequency.

5. Conclusion

In this study, we established the second harmonic vibration model. This model was verified by calculating $s_{11}^{E(2)}$ from the mode shape measurement and the velocity at the tip of the transducer. During the measurements, the temperature was kept constant to eliminate the effect of temperature increase. To introduce this nonlinear model to the FEM, the effect of the heat generation while driving should be considered in the next study.

References