A Modified Equivalent-Network Model for the Liquid-Level Sensors Operating in Trapped-Energy Vibration Modes

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1. Introduction

Novel sensors for detecting a small-scale variation in liquid level that employ a trapped-energy mode of conventional and/or backward-wave type have been studied by the authors’ group \(^1\)–\(^{10}\). The sensors have been modeled \(^6\)–\(^{10}\) by a distributed-constant electric network representing the propagation of thickness-vibration modes \(^11\),\(^12\). In these models the effect of liquid loading is expressed by putting the characteristic impedance of the corresponding part to be complex \(^{10}\). In this paper, an improved treatment is presented for the transmission-line model that employs a complex wavenumber for the liquid-immersed region, and some results of simulation are shown.

2. Geometry of the Sensors Utilizing Trapped-Energy Vibration Modes

The sensor configurations utilizing trapped-energy vibration modes are shown in Fig. 1 for a conventional type, (a), and a backward-wave type, (b). By dipping the evanescent-wave region of the resonators in a liquid, a depth-dependent variation in the electric admittance \(Y\) will occur at the resonance. In the case of backward-wave-type energy trapping \(^{12}\),\(^{13}\), the surrounding region of the piezoelectric plate is electroded and short-circuited as presented in Fig. 1(b). An additional capacitance \(C_A\) is connected in series with the central excitation electrodes to ensure energy trapping.

3. Equivalent-Network Modeling and Results of Analyses

The equivalent network model for the sensor utilizing the conventional trapped-energy mode is rather simple \(^5\). In the backward-wave-type trapped-energy vibrator \(^{10}\), however, there exists a non-electroded gap region between the central and the surrounding electrodes. The wavenumber there can be real even when the energy-trapping works. Therefore, the wavenumber and the characteristic impedance of this region should be complex to take the leakage loss into consideration when this region is immersed in a liquid.

Figure 2 shows the equivalent network model for the sensor utilizing the backward-wave-type energy trapping \(^9\),\(^{10}\). Here, the liquid surface may be either on the gap or on the outer electrodes. In the sensing side, two transmission lines representing the unelectroded gap of the length \(2l'\) are connected serially to the network elements corresponding to the central excitation electrode part. One is the transmission line representing the out-of-liquid portion of length \(2d\), where the wave number is \(\gamma'\) and the characteristic impedance is \(Z_0'\). The other is the line of length \(2l''\) \((-2l'-2d)\) representing the portion in the liquid. The wavenumber and the characteristic impedance in
this part are complex values and expressed as \( \gamma'' \) and \( Z_0'' \), respectively. Here, \( \gamma'' \) is expressed by introducing a factor \( m \) as:
\[
\gamma'' = \gamma'(1 - jm)
\]
The outermost metallized region is supposed to have an infinite length and is therefore expressed by the corresponding characteristic impedance \( Z_02 \).

A thickness-poled PbTiO\(_3\) plate is assumed as the backward-wave-type trapped-energy resonator model. The ratio of the central electrode width \( 2l \) to the plate thickness \( 2H \) is supposed to be 4.0 and the normalized gap width \( l'/H \) is supposed to be 0.5 or 1.0. The ratio of the damped capacitance \( C_0 \) to the series capacitance \( C_A \) is 1.0. A small amount of resistance is added at the electric port to take the material loss into account.

It is noted that continuous decrease in the electric conductance level is obtained as the liquid surface approaches to the central electrodes (\( d/H \) reduces to 0).

4. Conclusions

An improved treatment has been presented for the equivalent-network model of the liquid-level sensor proposed by the authors. Variation of the electric conductance on the liquid level presented in the former studies\(^{1-5}\) is well simulated. However, further investigation is required to clarify the relationship between the elastic property of the liquid and the factor \( m \).

References