Dispersion property of guided waves propagating a helical structure obtained by analysis and experiment

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1. Introduction

Guided wave ultrasonic modes propagating over long distances can be applied to non-destructive evaluation (NDE) for wire-ropes. Guided waves have a dispersion property, which represents frequency dependence on propagating velocities. Therefore, for effective use of guided waves, we need to know this property, which is usually expressed as dispersion curves [1]–[2]. Furuwasa et al. [3] remarked L-mode of guided waves have advantages for NDE.

Dispersion curves of complex structures including wire-ropes are generally obtained by a semi-analytical finite element (SAFE) method [2],[4]–[10], because the SAFE needs a smaller amount of calculation time and memory than the finite element method (FEM). However, if a cross-section is a semi infinity rectangle or a circle, a full analytical method can be developed even for complex geometries including helical structures.

Thus, the purpose of this study is to obtain dispersion curves of L-mode waves propagating a helical structure by a full analytical method. Obtained results are compared with experimental results.

2. Analysis

1. Governing equation

The Navier governing equations, which is the equation of motion for an elastic isotropic solid, are:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + 2\mu) \nabla \nabla \cdot \mathbf{u} - \mu \nabla \times \nabla \times \mathbf{u} \quad (1)$$

This equation can also be written in the cylindrical coordinate system. Guided waves along cylindrical structures propagate as harmonic waves expressed by,

$$u_r = U(r) \cos n\theta e^{i(kz-\omega t)} \quad (2.1)$$
$$u_\theta = V(r) \sin n\theta e^{i(kz-\omega t)} \quad (2.2)$$
$$u_z = W(r) \cos n\theta e^{i(kz-\omega t)} \quad (2.3)$$

Here we consider propagation of L-mode waves along a helical structure. Now, a new coordinate system (r, θ, s) is constructed from the orthonormal basis (N, B, T), for which a position vector \( \Phi(r, \theta, s) \) can be expressed as,

$$\Phi(r, \theta, s) = R(s) + N(s)r \cos \theta + B(s)r \sin \theta \quad (3)$$

Using the above Serret-Frenet formulation, it can be shown that this kind of mapping yields the following non-orthogonal covariant basis (\( \frac{\partial \Phi}{\partial r}, \frac{\partial \Phi}{\partial \theta}, \frac{\partial \Phi}{\partial s} \)), denoted by \( (g_1, g_2, g_3) \). Here, \( \kappa \) is the curvature and \( \tau \) is the torsion.

$$g_1 = \frac{\partial \Phi}{\partial r} = N(s) \cos \theta + B(s) \sin \theta \quad (4.1)$$
$$g_2 = \frac{\partial \Phi}{\partial \theta} = -N(s)r \sin \theta + B(s)r \cos \theta \quad (4.2)$$
$$g_3 = \frac{\partial \Phi}{\partial s} = T(s) + (\tau B(s) + \kappa T(s))r \cos \theta - \tau N(s)r \sin \theta \quad (4.3)$$

The covariant metric tensor, denoted by \( g_{ij} = g_i \cdot g_j \), and the Christoffel symbol are given by

$$g = \begin{bmatrix} 1 & 0 & 0 \\
0 & r^2 & \tau r^2 \\
0 & \tau r^2 & (\tau r)^2 + (1 - \kappa r \cos \theta)^2 \end{bmatrix} \quad (5)$$
$$\Gamma^k_{ij} = g_{ij} \cdot g^k \quad (6)$$

where the covariant basis \( (g^1, g^2, g^3) \) is defined by \( \delta^j_i = g^i \cdot g_j \).

The strain and stress tensors in the helical coordinate system can be described by using the displacement expressed in the cylindrical coordinate system with the Christoffel symbol as follows,

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) - \Gamma^k_{ij} u_k \quad (7)$$
$$T_{ij} = C_{ijkl} \epsilon_{kl} \quad (8)$$

Here the boundary condition is assumed to be stress-free

$$T_{rr}|_{r=a} = 0 \quad (9)$$
2. Analytically obtained dispersion curves

Fig. 1 shows analytically obtained dispersion curves of L-mode for experimental helical parameters. Black dots indicate dispersion curves for inner paths and blue dots indicate dispersion curves for outer paths in a helical volume.

3. Experiment

In our experiment, guided waves along a 1-meter aluminium helical specimen, shown in Fig. 2, were investigated with electromagnetic acoustic transducers (EMATs). The curvature of the specimen was 3.47[1/m], and its torsion was 0.897[1/m].

Experimental results are plotted by red circles in Fig. 1. They are roughly close to analytical results, though more delicate transmission and detection of waves may be required for higher accuracy.

4. Conclusions

An analytical method of calculating dispersion curves for helical structures has been developed by introducing the helical coordinate. Obtained results are roughly close to experimental results.

References


\[
T_{rz}|_{r=a} = 0 \quad (10)
\]

where \( a \) is the radius of the cross-section of the helical structure.

The determinant of the coefficient matrix appearing in equations (8) satisfying this boundary condition must be zero to have non-trivial solutions. This gives the dispersion equations in the helical coordinates.