Analysis of relation between liver fibrosis structure and fluctuation in co-occurrence matrix of ultrasonic images of fibrotic liver using multi-Rayleigh model
マルチレイリーモデルによる超音波画像の同時生起行列の揺らぎと線維構造の関係の解析

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1. Introduction

Although ultrasonic diagnostic equipment is widely used as a non-invasive and real-time imaging modality for liver disease, its outcome is dependent on the skill of the doctor. Thus, quantitative diagnostic method based on ultrasonic echo signal is highly required. As an ultrasonic image with a speckle pattern contains information on tissue structure, we have been analyzing co-occurrence matrices of ultrasonic images of fibrotic liver for quantitative tissue characterization1). In this report, we simulated B-mode images of different fibrotic liver models and examined quantitative relationship between co-occurrence matrix and fibrosis structure in combination with echo amplitude distribution analysis using multi-Rayleigh model.

2. Co-occurrence matrix and contrast

Co-occurrence matrix is one of the texture analysis methods. On an image \( f \), a pair of pixels with coordinates \((i, j)\) and \((m, n)\) are given by distance \( r \) and angle \( \theta \) between them as shown in Fig.1(a). Probability of the pair of pixels whose amplitudes are \( a \) and \( b \) is defined by following Eq.(1)

\[
P(a, b; r, \theta) = P(f(i, j) = a, f(m, n) = b)
\]

(1)

Given fixed positional parameters \( r \) and \( \theta \), co-occurrence matrix is generated and has the element of probability specified with amplitudes \((a, b)\) of the pixel pair as shown in Fig.1(b).

Texture feature contrast is used to quantify the distribution of the co-occurrence matrix and defined by Eq.(2).

\[
CNT(r, \theta) = \sum_{a=0}^{n-1} \sum_{b=0}^{n-1} (a - b)^2 P(a, b; r, \theta)
\]

(2)

Contrast value increases when the distribution spreads over the matrix and we expect that both amplitude and spatial information of the image can be extracted by analyzing contrast response against varying distance \( r \).

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3. Method and Results

3.1 Method of ultrasonic image simulation

For quantitative analysis, we used simulated ultrasonic images with scatterer distribution models of fibrotic liver and periodic fiber tissue in lateral direction as shown in Fig.2. In both models, scatterer density ratio of normal tissue and fibrotic tissue is set to 1:7 with the same fiber tissue rate (20%). Simulation ultrasonic images with 5MHz linear array transducer were created using Field II on MATLAB.

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Fig.1. How to make co-occurrence matrix: (a) relative position of two pixels and (b) display method.

Fig.2. Scatterer distribution models: (a) fibrotic liver and (b) periodic fiber tissue.

3.2 Theoretical value of contrast

When distance \( r \) becomes large enough against echo imaging resolution (Point Spread Function), the
spatial correlation between a pair of pixels is lost and its probability can be given by Eq.(3) using independent amplitude probability \( P(x) \)

\[
P(a, b; r, \theta) = P(a) \cdot P(b) \tag{3}
\]

In fibrotic liver, echo amplitude distribution is well expressed by multi-Rayleigh model consisting of Rayleigh distributions with different variances corresponding to fiber tissue and normal tissue respectively\(^2\). The theoretical contrast value at large distance \( r \) can be estimated from \( P(x) \) using multi-Rayleigh model\(^3\).

3.3 Analysis result

We calculated contrast values from co-occurrence matrices of ultrasonic images (Fig.3(i)) simulated with the models shown in Fig.2 and compared them with the theoretical values given by Eq (3). Results are shown in Fig.3(ii). Here angle \( \theta \) is fixed at zero (lateral direction) and distance \( r \) varies from 0 to 20 mm. Contrast of the fibrotic liver converges with the theoretical value but fluctuates around the theoretical value. Contrast of the periodic fiber tissue shows clearer fluctuation corresponding to the structural periodicity while its theoretical value is close to that of fibrotic liver.

To visualize what pixel pairs compose the fluctuation, we defined contrast delta as the difference between calculated value and theoretical value in the following equation,

\[
\Delta \text{CNT}(r, \theta) = \sum_{a=0}^{n-1} \sum_{b=0}^{n-1} [(a - b)^2 P(a, b; r, \theta) - (a - b)^2 P(a) \cdot P(b)]
\]

and mapped the elements of summation on the co-occurrence matrix. Fig.3(iii) shows the mapped elements at distance \( r_1 \) where contrast delta has positive peak and Fig.3(iv) shows those at distance \( r_2 \) where contrast delta has negative peak. Distribution of positive and negative elements are inhomogeneous and characteristic at \( r_1 \) and \( r_2 \) respectively, and has similarity between two tissue models. This means that in both tissue models the contrast fluctuation is produced by pixel pairs reflecting structural variation of the tissue in \( \theta \) direction.

4. Conclusion

We examined texture feature contrast calculated from co-occurrence matrices of simulated ultrasonic images for fibrotic liver and periodic fiber tissue models. Contrast plotted against varying distance \( r \) fluctuates around the theoretical value in both models and it is found that the fluctuation is produced by pixel pairs reflecting structural information of the tissue. As the next step, we plan to analyze the quantitative relationship between the fluctuation and parameters of fibrotic tissue structure.

References


Fig.3. Result of texture feature analysis for (a) fibrotic liver and (b) periodic fiber tissue: (i) simulated ultrasonic image, (ii) calculated contrast with theoretical value, (iii) contrast delta map at \( r_1 \), and (iv) contrast delta map at \( r_2 \).