Numerical Simulations of a Thermoacoustic Prime Mover at 20 kHz

20 kHzにおける熱音響の数値シミュレーション

Kyuichi Yasui†, Teruyuki Kozuka, Masaki Yasuoka, and Kazumi Kato
(National Institute of Advanced Industrial Science and Technology (AIST))

1. Introduction

Miniaturization of thermoacoustic system is required for some practical applications of the system such as for the usage of waste heat from some electrical circuits. Some research groups have already performed experiments using a high frequency sound such as in the kHz range in order to miniaturize the system because the resonator (pipe) length is smaller for higher sound frequency.

Flitcroft and Symko 1 made a thermoacoustic prime mover (sound generator) which operates at a tube are given by Eqs. (1) and (2) respectively was generated in the system. Furthermore, it has been suggested that the thermoacoustic functions given in Refs. 2.

In the present study, numerical simulations of a thermoacoustic prime mover at 20 kHz are performed by the transfer matrix method of Ueda 2, Eqs. (1) and (2) are expressed using a matrix as Eq. (3).

where the working gas moves, is the work flow at the higher temperature side of a stack (n=0 or x=0),  and  is a matrix defined in Ref. 2. Using Eq. (3), the sound pressure and the particle velocity at any point in a stack are calculated from those at the lower temperature side of a stack (n=0 or x=0).

where E is the unit matrix.

When there is negligible thermal conduction between a tube and the surroundings, the total enthalpy flux ( ) is conserved along a narrow tube. The work flow (acoustic power) (I) is calculated by Eq. (5).

where is the area of the cross section of a stack where the working gas moves, and are the phase angle of the acoustic pressure (P) and that of the particle velocity (U), respectively. The heat flux is calculated by Eq. (6).

Finally, two alternate definitions of the efficiency are discussed. The first definition is the rigorous physical definition as follows.

where is the efficiency, is the change in the work flow across a stack ( where and are the work flow at the higher and lower temperature side, respectively), and is the heat flux at the higher temperature side of a tube.

numbers including the information of their phases, is the position along the straight-tube with its origin at the lower temperature side of a stack, is the unit imaginary number, is the angular frequency of an acoustic wave, is the mean density, the mean pressure, the ratio of specific heats, and is the Prandtl number of the working gas, respectively. and are the thermoacoustic functions.

2. Model

In the present study, a stack is assumed to consist of a set of narrow straight tubes. Furthermore, it has been suggested that the capillary-tube-based thermoacoustic theory is also valuable in predicting the acoustic properties of random porous media such as a stack of aluminum foam. The momentum and continuity equations for a tube are given by Eqs. (1) and (2)

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The other definition ($\eta_{eng}$) is useful from the viewpoint of engineering:

$$\eta_{eng} = \frac{I}{Q}$$  \hspace{1cm} (8)

where $I$ and $Q$ is the spatial average of the work flow in stack and that of the heat flux, respectively.

### 3. Results and Discussions

In the present study, numerical simulations are performed for 20 kHz in sound frequency. The radius of a narrow tube in a stack is 0.02 mm. The other conditions are the same as those in the experiment of Ueda et al.\(^5\) The working gas is air at atmospheric pressure. The pressure amplitude of sound at $x=0$ is 3.4 kPa. For the work flow to increase, the total enthalpy flux should be larger than $30IC$ in magnitude. In Fig. 1, the result for $H_{flux}=-200IC$ is shown. There is a critical length in a stack above which the work flow begins to decrease. The reason of the decrease is the decrease of $\cos(\phi-\theta)$ due to the increase in $\phi-\theta$ with distance in a stack.

![Graph showing the result of the numerical simulation on the work flow as a function of $x$ in a stack.](image)

In Fig. 2, the results of the numerical simulations for various radii of a narrow tube on the energy efficiency defined are shown. The engineering efficiency (Eq. (8)) is always higher than the physical one (Eq. (7)). The physical one is as low as 0.1-0.5 % even at the optimum $\omega\tau_0$, where $\omega$ is the angular frequency of sound and $\tau_0$ is the thermal relaxation time at the lower temperature side of a stack ($x=0$)\(^7\).

![Graph showing the results of the numerical simulations on the energy efficiency.](image)

### 4. Conclusions

Numerical simulations of a thermoacoustic prime mover at 20 kHz have revealed that there is a critical length in a stack above which the work flow begins to decrease. The reason is the increase in the difference of phase angles between pressure and particle velocity of sound in a stack with distance. Two alternate definitions of energy efficiency are discussed.

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**References**