Sub-grid Technique for Numerical Simulation of Sound Wave Propagation Combining Constrained Interpolation Profile Schemes

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1. Introduction

To date, numerical analysis for sound wave propagation in time domain has been investigated widely as a result of computer development. Now, the development of accurate numerical schemes is an important technical issue[1].

The constrained interpolation profile (CIP) method, a novel low-dispersive numerical scheme is a type of method of characteristics (MOC) [2-4].

However, new grid systems are required for CIP simulations of complicated heterogeneous media or large-scale simulations of wave propagation. To overcome this problem, sub-grid techniques[5] are proposed for other simulation methods of wave propagation. In the previous study, we have introduced this technique for the type-C and type-M CIP methods, and evaluated the setting of the boundary interface between the course grid and sub grid[6].

The type-M CIP method is a simple technique with smaller memory use and less calculation time required than the type-C CIP method in exchange for accuracy. Therefore, from the point of reduction in the calculation cost, a sub-grid technique for the type-M CIP method is also important.

Subgrids are defined as those smaller than the surrounding grids: we can use suitable multisize grids in an analysis domain according to a sub-grid technique for the CIP-MOC simulation of sound wave propagation.

In this study, we have improved on the sub-grid techniques[6] for CIP analysis using generalized CIP (GCIP) schemes[7] and reported the comparison of accuracy and calculation cost.

2. Sub-grid techniques in CIP method

In CIP analysis, the governing equations for linear acoustic fields (a lossless medium) are transformed into advection forms. For example, for the calculation of $x$-advection, the advection equation is given as

$$\frac{\partial(p \pm Zv)}{\partial t} \pm c \frac{\partial(p \pm Zv)}{\partial x} = 0. \quad (1)$$

In this equation, $p$ is the sound pressure, $v_x$ is the particle velocity, $Z$ signifies the characteristic impedance (i.e. $Z = \sqrt{\rho K}$) and $c$ represents the sound velocity in medium (i.e. $c = \sqrt{K/\rho}$). Here, $\rho$ denotes the density of the medium, and $K$ represents the bulk modulus.

In addition, through simple spatial differentiation of the equations, the equations of the derivatives are given as

$$\frac{\partial(p \pm Zv)}{\partial t} \pm c \frac{\partial(p \pm Zv)}{\partial x} = 0. \quad (2)$$

Figure 1 shows the sub-grid technique in the CIP method. Here, $\Delta x$ and $\Delta y$ represent the course grid size, while $\Delta x'$ and $\Delta y'$ are sub grid size, respectively.

Figure 2 shows the treatment of the boundary course grid and the sub-grid in propagation of $\pm x$ direction. In first step, we interpolate $P$, $v_x$, $\partial P$ and $\partial v_x$ in direction using Hermite interpolation. Next, we calculate advection equations (Eqs.(1) and (2)) in $\pm x$ direction. Notice that sub grid technique in the CIP analysis just needs to change interpolating function in sub grid region, because CIP scheme is based on a two-point stencil’s MOC.

In this study, we use the GCIP scheme; GCIP(7,1), GCIP(3,1), and GCIP(3,0). Of these schemes, GCIP(7,1) and GCIP(3,0) respectively employ 7th-order Hermite interpolation and 3rd-order Lagrange interpolation with four stencils for the advection calculation.

Fig. 1 Sub-grid technique in the CIP method.
3rd order Hermite interpolation in y-axis (first step)

3rd order Hermite interpolation in x-axis (second step)

Fig. 2 Treatment of the boundary.

(a) $t = 10 \Delta t$
(b) $t = 500 \Delta t$
(c) $t = 1000 \Delta t$

Fig. 3 Distribution of the sound pressure

Fig. 4 Absolute pressure value: $|p_{\text{fine}}^t|$ and $|p_{\text{sub}}^t - p_{\text{fine}}^t|$ 

4. Results and discussion

We present numerical results obtained using the sub-grid technique in the CIP analysis. Calculation parameters are the following: the direction of acoustic field propagation, $x, y$ (two-dimensional analysis); course grid size, $\Delta x = \Delta y = 0.06$ m; sub grid size, $\Delta x = \Delta y = 0.02$ m; time step, $\Delta t = 3.79 \times 10^{-5}$ s; $\rho = 1.21$ kg/m$^3$ and $K = 1.42 \times 10^5$ Pa.

We also investigated the calculation time required for some sub-grid models. Here, we use a PC with Intel Core i7-980X Extreme Edition 3.33GHz. This processor has 6 cores and 12 hyperthreaded cores, or effectively scales 12 threads. For all analyses, parallel computation using OpenMP was applied.

Figure 3 shows the sound pressure distribution obtained using type-M CIP analysis with sub-grids at $t = 10 \Delta t$, $t = 500 \Delta t$ and $t = 1000 \Delta t$. Here GCIP(7,1) and GCIP(3,0) schemes are utilized for the advection calculation in coarse grids. The input pressure is driven from inside of the sub-grids. Here, the meshed area is the sub-grid region. We can ascertain the propagation behavior including that in the sub-grid region.

Figure 4 showed the error using sub-grids by means of comparison of the absolute pressure value at point A (see Fig. 3). We also show the numerical results obtained using the sub-grid technique for type-M CIP (i.e., GCIP31) analysis. Calculation parameters of both analyses are on equal terms. It is confirmed that the boundary in the sub-grids has good permeability characteristics with low reflection. The numerical error of the type-M GCIP(7,1) method is a little smaller than that of the type-M CIP method for acoustic simulation with a subgrid system.

Figure 5 shows the comparison of calculation time, where the calculation is divided into 500 time steps. Table 1 is the calculation parameter. The sub-grid model has a much shorter calculation time than the fine grid model. Fig. 5 also shows that CIP analysis with course grid that calculates with 7th-order Hermite interpolation required more calculation time than 3rd-order Hermite interpolation. This was because the number of variables for course grid is different.

References