1. Introduction

The Synthetic Aperture (SA) array signal processing is an attractive tool to realize ultra-high framerate ultrasonic imaging and tissue dynamic property detection for quantitative analysis of various tissue oriented characteristics in-vivo. For fully utilizing the flexibility of the ultrasonic arrayed transducer Range Stacking (RS) algorithm is still an active tool of Synthetic Aperture Radar (SAR) signal processing [1,2] in spatial frequency domain toward extension of target area to be measured. This algorithm can be straightforwardly applied to SA array signal processing of successive ultrasonic echo frames which could be obtained by pulsed irradiations of plane wave transmitted from an array transducer. However we have been studying to estimate overall system performances of SA array processing controlled by localization and power of a point irradiation source generation for accurate and ultra-high framerate measurement [3-6]. Herein the passive (one-way receiving) SA algorithm in spatial frequency domain is modified for spherical transmission and its overall system performance is quantitatively evaluated as a practical method to extend the targets area to be measured.

2. Passive Range Stacking Algorithm

The Range Stacking algorithm for successive calibration of range dependent point spread function in ordinary SA signal processing is described by spectrum $S(\omega, k_n)$ of 2D echo signal $s(t, u)$, where $\omega$ and $k_n$ are angle frequency on delayed time $t$ from a pulsed transmission $p(t)$ and spatial frequency on lateral range $u$ along an arrayed aperture, respectively [1,2]. As a simplified case the passive array Range Stacking is performed by superposing each successive reconstruction at the range bins $x_n$'s over the total measurement range with a fixed resolution, as shown in Fig. 1.  

$$S(\omega, k_n) = P(\omega) e^{-i\sqrt{k_n^2 - k_n^2} x_n - jk_n Y_n} \Gamma(\sqrt{k_n^2 - k_n^2}, k_n),$$

where $P(\omega)$ and $\Gamma(k_\omega, k_n)$ are the spectrum of a transmitting pulse $p(t)$ and the 2D Fourier-transform of $\gamma(x, y)$ which is located with the offset from the target center $(X_C, Y_C)$, respectively. Consecutively the spectrum is matched-filtered with the following spectrum $S_{0\omega}(\omega, k_n)$ of the reference signal $s_{0\omega}(t, u)$ at the range bin $x_n$.  

$$S_{0\omega}(\omega, k_n) = P(\omega) e^{-i\sqrt{k_n^2 - k_n^2} (X_C + x_n) - jk_n Y_n}.$$

The result is integrated over the available frequency band with the assignment of $k_y = k_n$ and inversely Fourier-transformed with respect to $k_y$. Thus the target function $\gamma(x_n y)$ at the range $x_n$ is formulated as follows,
The total target function is obtained by superposing the local reconstruction at the range $x_n$ successively.

### 3. Extended Range Stacking Algorithm

When utilizing the active spherical source generated by each arrayed element in transmitting phase the Range Stacking algorithm has to be extended to account for the time delay that the pulsed irradiation takes to arrive at each target with the different cross range in the same range bin from that of the spherical source, as shown in Fig. 2.

Thus, we propose the extended Range Stacking algorithm which includes the modified 2D matched filter $S^*_0(x_n) S^*_{0x_n} (\omega, k_n) e^{i\omega k_n y} dk_n$ where $c$ is sound velocity.

$$
\gamma(x_n, y) = \int \int S(\omega, k_n) S^*_0(x_n) S^*_{0x_n} (\omega, k_n) d\omega e^{i\omega k_n y} dk_n
$$

in which the re-sampled cross range $y_m$ is obtained from the following parabolic curb of the time delay vs. $y_m$ under the point source location $(X_T, Y_T)$.

$$
m\Delta t = \left( (X_C + x_n - X_T)^2 + (Y_C + y_m - Y_T)^2 \right) / c
$$

where $c$ is sound velocity.

### 4. Simulation Results

The modified Range Stacking algorithm was numerically evaluated for the same parameters as the experimental SA array signal processing system. The linear array transducer is having 256 elements with 0.25 mm pitch, 3 MHz center frequency and 2 MHz bandwidth. The fine reconstructions throughout the extended target area of 40mm by 32mm are depicted for the different point source irradiations, as shown in Fig. 3.

**Fig. 3** The SA reconstructed target functions by utilizing the extended Range Stacking array processing for each symmetric point source location at (8mm, 16mm) and (8mm, -16mm) as in the left and the right image, respectively.

### 4. Discussion

The proposed Range Stacking algorithm in SA array signal processing by point source generation contributes to the stable extension of target area with controllable resolution, in which the overall optimal system design can be quantitatively pursued in measurement accuracy and statistical variance for the evaluation of 2D motion vectors in-vivo.

### References