Comparison of Spatial Resolution in Parallel Beamforming and Diffraction Tomography

並列ビーム形成法と回折トモグラフィー法の空間分解能 の比較

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1. Introduction

Echocardiography is a predominant modality for diagnosis of the heart by non-invasive and real-time observation of cross-sectional images. In addition, it has been recently shown that measurement of rapid transition of myocardial contraction/relaxation and propagation of heart-wall vibration would be useful for assessment of myocardial function and viscoelasticity. However, such measurements require a higher frame rate of several hundred Hz. Therefore, we realized high frame rate echocardiography using parallel beamforming with unfocused beams [1]. On the other hand, there is another method for imaging, namely, diffraction tomography [2]. It was reported that diffraction tomography realized high temporal and spatial resolutions simultaneously. One of the problem of parallel beamforming with unfocused transmit beams is degradation of spatial resolution, and diffraction tomography may improve this problem. In this report, diffraction tomography is applied to high frame rate echocardiography with unfocused transmit beams, and spatial resolutions realized by diffraction tomography and parallel beamforming were compared.

2. Principles

By assuming that scattering coefficient $\gamma(\mathbf{r})$ at each spatial point $\mathbf{r} = (x, z)$ in biological tissue is very small, multiple scattering can be neglected (Born approximation). Under such condition, frequency spectrum $s(x_m, \omega)$ of signal received by *m*-th transducer element is expressed as follows [3]:

$$s(x_m,\omega) = k^2 \iint_{-\infty}^{\infty} p_i(\mathbf{r}) \cdot \gamma(\mathbf{r}) \cdot g(\mathbf{r}_m \mid \mathbf{r}) dx dz \qquad (1)$$

where x_m , ω , and k are lateral position of the *m*-th transducer element, angular frequency, and wavenumber, respectively. In the case of plane wave transmission (direction: (q_x, q_z)),

$$p_i(\mathbf{r}) = A \cdot \exp\{j(q_x x + q_z z)\}.$$
 (2)

By assuming that the scattered wave is a circular wave, Green's function $g(\mathbf{r}_m | \mathbf{r})$ ($r_m = (x_m, 0)$) is expressed by the Hankel function of the 0-th order and the first kind as follows:

$$g(\mathbf{r}_{m} | \mathbf{r}) = H_{0}^{(1)}(k | \mathbf{r}_{m} - \mathbf{r})$$

= $\frac{j}{4\pi} \int_{-\infty}^{\infty} \frac{\exp\left[j\left\{k_{x}(x_{m} - x) - \sqrt{k^{2} - k_{x}^{2}}z\right\}\right]}{\sqrt{k^{2} - k_{x}^{2}}} dk_{x},$
(3)

By substituting Eqs. (2) and (3) into Eq. (1) and taking Fourier transform of Eq. (1) with respect to x_m , 2-D frequency spectrum $S(k_x, \omega)$ of signals received by the transducer array is expressed as follows:

$$S(k_x, \omega) = \frac{jk^2 A}{2\sqrt{k^2 - k_x^2}} \int_{-\infty}^{\infty} \gamma(x, z)$$

$$\times \exp\left[-j\left\{(k_x - q_x)x + \left(\sqrt{k^2 - k_x^2} - q_z\right)z\right\}\right] dxdz \quad (4)$$

The right-hand side of Eq. (4) corresponds to 2-D frequency spectrum $\Gamma(k_x - q_x, (k^2 - k_x^2)^{1/2} - q_z)$ of scattering coefficient $\gamma(x, z)$.

$$\Gamma\left(k_{x}-q_{x},\sqrt{k^{2}-k_{x}^{2}}-q_{z}\right)$$

= $-j\frac{2\sqrt{k^{2}-k_{x}^{2}}}{A\cdot k^{2}}\cdot S(k_{x},\omega).$ (5)

By taking 2-D inverse Fourier transform of Eq. (5), the spatial distribution of scattering coefficient $\gamma(x, z)$ is estimated.

3. Basic Experiment Using Phantom

In the present study, a commercial ultrasound diagnostic system (α -10, Aloka), which was modified so as to acquire ultrasonic echoes received by individual transducer elements, equipped with a 3.75-MHz phased array probe (UST-52101, Aloka) was used. In this basic experiment, an ultrasound imaging phantom (model 54GS, CIRS) was used to evaluate the spatial resolution of an ultrasound image.

Figures 1 and 2 show results obtained by

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parallel beamforming and diffraction tomography, respectively. Both results were obtained by only one transmission of a plane wave (steering angle: 0 degrees). (a) in each figure shows a B-mode image, and (b), (c), and (d) show lateral amplitude profiles at depths of about 44 mm, 64 mm, and 84 mm, respectively. Only in Fig. 1(b), amplitude profiles obtained by rectangular apodization (red plots and line) and Hamming apodization (green plots and line) are shown. As shown in Figs. 1(b) and 2(b), the amplitude profile obtained by diffraction tomography corresponds to that obtained by parallel with beamforming rectangular apodization. Therefore, results obtained by parallel beamforming with rectangular apodization and diffraction tomography were compared.



Fig. 1 Experimental results obtained by parallel beamforming. (a) B-mode image of phantom. (b) Lateral amplitude profile at a depth of about 44 mm (red plots and line: rectangular apodization, green plots and line: Hamming apodization). Amplitude profiles at depths of about (c) 64 mm and (d) 84 mm.

Spatial resolution of each image was evaluated by calculating the widths at half maximum of the amplitude profiles as shown in Table 1. As shown in Table 1, spatial resolutions of parallel beamforming and diffraction tomography are similar in the shallow region. However, diffraction tomography realizes better spatial resolution in the deeper region compared to parallel beamforming.



Fig. 2 Experimental results obtained by diffraction tomography. (a) B-mode image of phantom. Lateral amplitude profiles at depths of about (b) 44 mm, (c) 64 mm, and (d) 84 mm.

Depth	44 mm	64 mm	84 mm
PBF	1.18 mm	1.79 mm	2.27 mm
DT	1.16 mm	1.70 mm	1.90 mm

Table 1 Comparison of lateral spatial resolutions of parallel beamforming (PBF) and diffraction tomography (DT).

6. Conclusion

In this study, spatial resolutions of parallel beamforming and diffraction tomography were compared. Diffraction tomography realized better spatial resolution in the deeper region.

References

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