# Measurement of Liquid Viscosity near a Wall Using Optoacoustic Interaction

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## 1. Introduction

Viscosity measurement is used for monitoring process of chemical reaction and degradation of lubricating oil. As a method of viscosity measurement, rotational viscometers are commonly used.<sup>1)</sup> However, these viscometers need inserting probes into target liquid to measure viscosity. We have been studied for viscosity measurement method with ultrasonic wave. Viscous stress is strongly applied to the fluid particle near the wall, and the wavefront of propagating sound slants. The wavefront slope near a wall differs by the viscosity of medium. Thus, it is considered that the viscosity can be determined by measuring the wavefront slope. Schlieren method can be employed to observe sound wavefront.<sup>2)</sup> However, the method requires stroboscopic measurement and high-speed camera. We focus on light diffraction. An incident laser to sound wave with short wavelength causes diffracted light normal to sound wavefront.<sup>3)</sup> The wavefront slope can be measured by calculating the slope of diffracted light with CCD camera in far-field. This method is more simple than Schlieren method because it uses only a laser system, CCD camera, and lens.

In this paper, the sound propagation near the wall in viscous liquid is simulated. Then, the phase grating formed by the sound, and the diffraction pattern by it is calculated. The simulations are performed assuming that the water and epoxy resin as low-viscous and high-viscous liquid, respectively. The wavefront slope by the viscosity of the medium liquid is compared to confirm the difference in the rotation angle of the diffraction lights by the slope of sound wavefront.

## 2. Simulation of Viscosity Measurement

## 2.1 Sound wave propagation simulation

Particle behavior in viscous liquid is described by using equation of continuity and Navier-Stokes equation as;<sup>4)</sup>

$$\frac{\partial V}{\partial t} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \nabla^2 V, \qquad (1)$$

$$\frac{\partial p}{\partial t} + K \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 , \qquad (2)$$



Fig. 1 Simulation setup for sound wave propagation in liquid.

**Table I** Liquid parameters for simulation of soundwave propagation.

	Sound velocity	Density	Viscosity
	<i>c</i> ( m/s )	$\rho (kg/m^3)$	$\mu$ (mPa s)
Water	1500	1000	1
Epoxy resin	1650	1160	3200

where,  $\rho$ , p,  $\mu$ , and K are density, sound pressure, viscosity, and bulk modulus, respectively. V = (u, v) is particle velocity vector, where u and v are the x- and y- components of the vector, respectively. Eqs. (1) and (2) are discritized and solved to obtain the particle velocity and the sound pressure by Finite-Difference Time-Domain (FDTD) method.

**Figure 1** shows simulation setup. The sound pressure was applied on the *yz* plane at x = 0 mm. Rigid wall condition was applied on the *xz* plane at y = 0 mm. Mur absorbing boundary condition was applied on the *xz* and *yz* planes at x = 5 and y = 3(mm), respectively. Ultrasound frequency was 5 MHz. Time step and mesh size was set as 2 ns and 0.01 mm, respectively. The parameters of mediums are shown in **Table I**.

2.2 Light diffraction simulation

The phase of the light in sound field is shifted by sound pressure. The degree of the phase shift is called Raman-Nath parameter, and it is calculated as;

$$\zeta = \frac{2\pi L}{\lambda} \frac{dn}{dp} p \,. \tag{3}$$

where,  $\zeta$ , dn/dp, L, and  $\lambda$  are the Raman-Nath parameter, acousto-optic coefficient, the propagation distance of light in ultrasound, and the wave length of the light, respectively. The diffraction pattern can be calculated by two-dimensional Fourier transform to the optical phase shifted light.

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Figure 2 shows light diffraction simulation setup. Laser beam entered the liquid. The incident position of the laser beam was (x, y) = (2.5, 0.1)(mm). Amplitude of the laser beam, l(x, y), was described as;

$$l(x, y) = \exp\left(-4\frac{(x-2.5)^2 + (y-0.1)^2}{2d^2}\right), \quad (4)$$

where d was the diameter of the laser beam and set as d = 0.8 mm. In the liquid, the optical phase of laser is shifted by the sound pressure distribution. The sound pressure distribution, p(x, y), was calculated as described in §2.1. The phase shift grating,  $\phi(x, y)$ , was described as;

$$\phi(x, y) = \zeta(x, y) = \frac{2\pi L}{\lambda} \frac{dn}{dp} p(x, y).$$
(5)

The sound pressure distribution continued from z = 0 to 30 (mm), and L,  $\lambda$ , and the peak value of the sound pressure were set as 30 mm, 632.8 nm, and 3 kPa, respectively. Acousto-optic coefficient, dn/dp, of water and epoxy resin was  $1.47 \times 10^{-10}$  Pa<sup>-1</sup>. Considering above, the optical phase shifted light, i(x, y), was described as;

$$i(x, y) = l(x, y) \exp(j \cdot \phi(x, y)), \qquad (6)$$

where *j* was imaginary number. In order to simulate light diffraction, two-dimensional fast fourier transform (FFT) was employed. The diffraction pattern on the observation plane,  $H(v_x, v_y)$ , was obtained as;

$$H(v_x, v_y) = FFT(h(x, y)^2), \qquad (7)$$

where  $(v_x, v_y)$  were x- and y- components of number of waves, respectively. The zeroth-order diffraction light was observed at center of the observation plane, and the first-order diffraction lights were observed at the both sides of the zeroth-order diffraction light.

#### 3. Results and Discussions

Figures 3(a) and 3(b) show sound pressure distributions in water and epoxy resin, respectively. While the wavefront in water did not slant near the rigid wall, that in epoxy resin slanted in the range of y = 0 to 0.5 (mm).

Figures 4(a) and 4(b) show diffraction patterns by sound pressure distribution in water, and in epoxy resin, respectively. To confirm difference in Figs. 4(a) and 4(b), the slopes between two points, that have the maximum intensity in the first-order diffraction lights in each image, were calculated. As a result, the rotation angle in Figs. 4(a) and 4(b) were 0 and 0.05 (rad), respectively. This result indicates that the first-order diffraction lights rotate when the light passed through the sound field in which wavefront was slanted by the viscosity of the medium.



Fig. 2 Simulation setup for light diffraction by optoacoustic effect.



**Fig. 3** Sound pressure distribution; (a) in water and (b) in epoxy resin.



**Fig. 4** Diffraction pattern by sound pressure distribution; (a) with water and (b) with epoxy resin.

## 4. Conclusion

The purpose of this paper is to measure viscosity by measuring the rotation angle of diffracted lights by sound wave. The light diffraction by sound field in two kinds of liquids, water and epoxy resin, was simulated. As a result, it was indicated that the sound wavefront slope differs by the viscosity of liquid, and the slope causes the rotation of the diffraction pattern. Thus, viscosity can be measured by observing the slope of diffraction light in medium. Experiments are planned to confirm that the sound wave in viscous liquid corresponds to the theory.

#### References

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