# Quantitative analysis of nonlinear ultrasonic response at closed cracks by the damped double node model

減衰2重節点モデルによる閉口き裂の非線形超音波応答の 定量的解析

Kentaro Jinno<sup>1†</sup>, Azusa Sugawara<sup>1</sup>, Koji Takahashi<sup>1</sup>, Yoshikazu Ohara<sup>1</sup>, and Kazushi Yamanaka<sup>1</sup>(<sup>1</sup>School of Eng., Tohoku Univ.)

神納 健太郎 <sup>1†</sup>,菅原 あずさ <sup>1</sup>,高橋 恒二 <sup>1</sup>,小原 良和 <sup>1</sup>,山中 一司 <sup>1</sup> (<sup>1</sup>東北大 工)

## 1. Introduction

Evaluation of closed crack is most serious issue for safety of important structure such as power plants or air planes. To solve this problem, nonlinear ultrasound, in particular subhamonics [1] is promising. So, simulation model for optimizing inspection condition is necessary. One dimensional model using damping term (e.g. [1]) and thin delamination model using double node[2] are reported. But, they are not enough for practical design of testing equipment for deep crack in thick object. So, we have reported 2D analysis of subharmonic generation at deep closed cracks using damped double nodes (DDN) by finite-difference time-domain (FDTD) method[3]. In this report, we extended the DDN model to real testing condition and demonstrated its applicability as simulation model of nonlinear ultrasonic images.

### 2. Concept of damped double node (DDN) model

In the open state, the normal nodes are split to double nodes consisting of the particle velocity of incidence-side crack face  $\dot{u}^-$  and particle velocity of transmission-side crack face  $\dot{u}^+$ . To simulate closed crack faces with compression residual stress, we introduced a viscous damping to suppress the noise[3].

(1) At the closed state, the tensile stress  $T_{IM}$  at the crack is calculated as the average of the stress of the left and right nodes of i-1 and i, such that

$$T_{1M} = \frac{1}{2} \left\{ T_1^{n}(i,j) + T_1^{n}(i-1,j) \right\}. \qquad \cdots (1)$$

If  $T_{1M} \leq T_{th}$ , the nodes remain close and if  $T_{1M} > T_{th}$ , the nodes are opened, where  $T_{th}$  is the compression residual stress giving the threshold for transition.

(2) At the open state, the particle velocity nodes  $\dot{u}^+$  and  $\dot{u}^-$  have viscous damping proportional to the particle velocity difference between  $\dot{u}^+$  and  $\dot{u}^-$ , so that

$$\dot{u}^{+n+1} (i,j) = \dot{u}^{+n} (i,j) + 2V_{PL}T_1^n(i,j) \dots(2) - \gamma \left\{ \dot{u}^{+n} (i,j) - \dot{u}^n (i+1,j) \right\}$$

$$\dot{u}^{-n+1} (i,j) = \dot{u}^{-n} (i,j) + 2V_{PL}T_1^n(i-1,j) \dots(3) - \gamma \left\{ \dot{u}^{-n} (i,j) - \dot{u}^n (i-1,j) \right\}$$

where  $\gamma$  is the damping coefficient, and V<sub>PL</sub> is the courant factor. Displacements are given as an integral of particle velocity  $\dot{u}^+$  and  $\dot{u}^-$ , respectively. The crack opening displacement (COD) is given as

$$\Delta u^n = u^{+n} - u^{-n} \qquad \cdots (4).$$

If  $\Delta u > 0$ , the crack remain open, and if  $\Delta u \le 0$ , the crack is closed.

# **3-1. Experimental and Simulation condition**

**Fig.1** shows experimental setup of A7075 fatigue crack specimen. The PZT array has 32 elements with center frequency of 5MHz. Incident burst wave is 7 MHz with 3 cycles, and incident angle  $\theta$  is 30° where the focal point was upper crack tip and 20° where the tip was out of focus.

Then, simulation was carried out by assuming the experimental set up. The time interval is 1.605 ns and node interval is 0.02 mm in the model. Residual compression stress was set to 100 MPa. The damping coefficient was set to 0.3.



#### 3-2. Simulation results of wave propagation

**Fig. 2** shows FDTD simulation image with two kinds of incident angle  $\theta$ . At  $\theta=20^{\circ}$ , the crack was still closed and only scattered wave from the notch was observed, because tensile stress  $T_{IM}$  was 33 MPa which is less than residual stress. At  $\theta=30^{\circ}$ ,  $T_{IM}$  was 287 MPa which is larger than residual stress. So, the crack was opened and scattered wave from the upper crack tip was observed.



(2-a)  $\theta$ =20° (2-a)  $\theta$ =30° **Fig.2** Snapshots of wave propagation

#### 3-3. Comparison of received waveforms

Those scattered wave were received by the array transducer. **Fig. (3-a)** shows received waveform of #16 element in simulation. At  $\theta$ =20°, only notch response was observed. On the other hand, at  $\theta$ =30°, the upper crack tip response was observed. Furthermore, there was a crack face response which observed between the upper crack tip and notch. And, it seems to have a lower frequency component than fundamental frequency (7 MHz).

**Fig.(3-b)** shows experimental results of received waveforms at #16 element. Experimental and simulation result were very similar. At  $\theta=20^{\circ}$ , crack response was not observed similarly to the simulation result. And at  $\theta=30^{\circ}$ , an upper crack tip response and the crack face response at lower frequency were observed. Moreover, the crack face response has lower frequency component than simulation result.

Then by using SPACE[4] algorithm, we created subharmonic array (SA) images by filtering at 2.5~4.5 MHz. **Fig.4** shows simulation and experimental results of SA images at  $\theta=30^{\circ}$ . In simulation result (4-a), not only the upper crack tip but also the middle part were observed, which can be expected by received waveform. These were very similar to the experimental result (4-b).

### 4. Discussion

Although closed crack face was completely flat in the simulation, middle part response was observed. Simulation SA image (4-a) indicates crack face scattering at middle part of the flat face. Though a crack tip causes scattering in linear ultrasounds, crack face does not usually causes scattering but only reflection. Therefore the crack face scattering is a new concept proposed in this work, that could explain the response at crack middle part in experimental SA image (4-b).



Fig.3 Received waveform of #16 element



**Fig.4** SA images (2.5~3.5 MHz)

### 5. Conclusion

By making simulation condition of DDN model similar to real fatigue crack specimen, simulation and experimental results were compared.

The simulation waveforms could reproduce the incident angle dependence. And, the upper crack tip and the crack face response were observed when incident ultrasound was focused at the upper tip.

The crack middle part response was observed both in simulation and experimental SA images. So, simulation result indicates crack face scattering.

Therefore, this model reproduced feature of experimental waveforms and images. So, DDN model, improved to real condition is useful in nonlinear ultrasonic crack evaluation.

#### References

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