

## Design of Liquid Density and Viscosity Sensor Using Dual Mode Rhombus Resonator

二つの振動モードを有する菱形振動子を用いる液体の密度と粘度を測定するセンサの設計

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### 1. Introduction

The density and the viscosity of liquid are important parameters for process monitoring and quality control in various industrial fields, such as food and medical engineering. To measure the density and the viscosity of liquid, sensors using piezoelectric resonator have been studied actively<sup>1-3</sup>. The authors have proposed a density and viscosity sensor using a single piezoelectric resonator<sup>4,5</sup>. In this sensor, the density and the viscosity of liquid are determined by measuring the changes of frequencies of two vibration modes of the sensor, and it has cost advantage compared to other sensors which measure resonant resistance and resonance frequency because the equipment to measure such factor becomes complicated. However, presented sensor has an issue; there is a large difference in the sensitivities of the two vibration modes, which results in limitation of the measurement range. In this paper, we propose rhombus shape of resonator to expand the range of measurement by making the sensitivity of two vibration modes in the same order.

### 2. Design of Sensor

The resonator of the sensor is composed of a piezoelectric resonator with rhombus bimorph structure and a copper plate as shown in Fig. 1. The copper plate is partially immersed in liquid. The tip of the resonator vibrates in tangential or normal direction with respect to the contact surface by driving the electrodes selectively as shown in Fig. 2. The following equation expresses the motion equation of the elastic plate in the liquid,

$$[K - \omega^2(m_e + m_l)]u + j(b_e + b_l)\omega u = F_0 e^{-j\omega t}, \quad (1)$$

where  $F_0$ ,  $u$ ,  $m_e$ ,  $b_e$ , and  $K$  are the amplitude of external force, displacement of the tip of elastic plate, equivalent mass of the resonator at the tip, the me-

chanical resistance, and spring constant, respectively. Moreover,  $m_l$  and  $b_l$  are mass loading and damping by the liquid, respectively. The resonance frequency when the tip is immersed in the liquid,  $f_{Liq}$ , and in the air,  $f_{Air}$ , in eq. (1) become,

$$f_{Liq} = \frac{1}{2\pi} \sqrt{\frac{K}{m_e + m_l}}, \quad f_{Air} = \frac{1}{2\pi} \sqrt{\frac{K}{m_e}}, \quad (2)$$

and  $K$  could be eliminated as,

$$\frac{f_{Liq}}{f_{Air}} = \sqrt{\frac{m_e}{m_e + m_l}}. \quad (3)$$

Regardless to the shape of resonator,  $f_{Air}$  and  $f_{Liq}$  have a relation of eq. (3). The density and the viscosity are derived from  $m_l$  at two vibration modes<sup>4-6</sup>, by measuring resonance frequency in the liquid. The previously proposed sensor has a rectangular shape of resonator. However, it was found that the frequency change in two vibration modes remain in the different order (Table 1), which results in difference of the sensitivities in two vibration modes. The liquid density and viscosity are shown in Table 2. This is because  $m_e$  in normal vi-

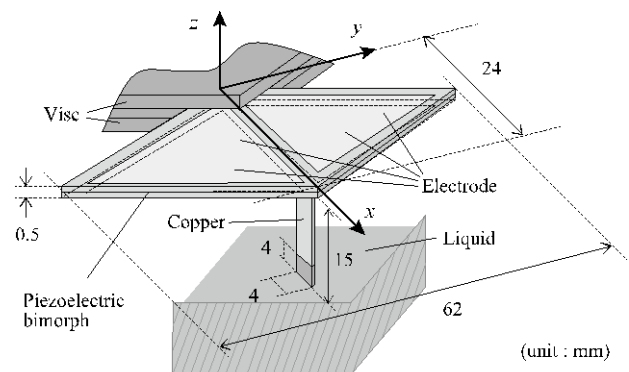


Fig.1 Schematic of proposed sensor.

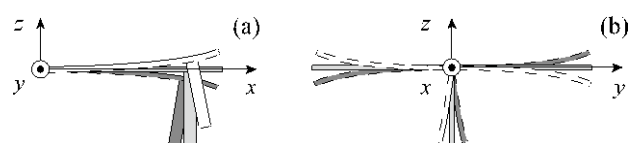


Fig.2 Direction of vibration of the sensor. (a) tangential vibration, and (b) normal vibration.

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bration is larger than  $m_e$  in the use of a rectangular shape of resonator. In this paper, we propose the rhombus shape of resonator to solve above problems. By employing rhombus resonator, the frequency change in two vibration modes, is especial to approach each other.

### 3. FEM analysis

#### 3.1 Analysis of equivalent mass

Figure 1 shows the sensor that was designed in Sec. 2, and the resonator was divided into 18,100 elements. The  $m_{te}$  and  $m_{ne}$ , equivalent masses of the resonator in tangential and normal vibration modes, are presumed as additional weight on the tip, whose mass is known. Before analyzing change of eigen frequency of the proposed resonator when the tip is immersed in the liquid, we analyzed the equivalent mass,  $m_{te}$  and  $m_{ne}$ , as preparation using Finite Element Method (FEM). **Table 3** shows  $m_{te}$  and  $m_{ne}$  of the proposed and the previous sensor. It is calculated from Table 3 that  $m_{te}/m_{ne}$  of the proposed sensor is 0.60,  $m_{te}/m_{ne}$  of the previous sensor is 0.31. Consequently, the proposed sensor has a better balance of  $m_{te}$  and  $m_{ne}$  than the previous sensor. Because when  $m_{te}/m_{ne}$  is closer to 1, the sensitivities of two vibration modes approach each other. This is caused by that normal vibration could be excited easily by employing rhombus shape of resonator.

#### 3.2 Analysis of eigen frequency in the liquid

Assuming that the tip of the sensor was immersed in the liquid, resistance stress were set as inertia resistance of liquid in tangential vibration and as viscous resistance of liquid in normal vibration, on the tip to each two vibration modes, respectively<sup>4,5</sup>. Table 1 shows eigen frequency changes of the sensor in the air and in the liquid of in two vibration modes, where  $\Delta f$  is  $f_{Air} - f_{Liq}$ . Changes of eigen frequency of the proposed sensor in the air and in the liquid of two vibration modes are closer than that of the previous sensor. We compare the sensitivity of the previous and the proposed sensor using  $r$  as,

$$r = \frac{\Delta f_{tan}}{\Delta f_{nor}} = \frac{f_{tAir} - f_{tLiq}}{f_{nAir} - f_{nLiq}} . \quad (4)$$

When  $r$  is closer to 1, sensitivity of frequency is close to in each vibration mode.  $r$  of previous sensor is 0.0210, while  $r$  of proposed sensor is 0.2136.  $r$  of proposed sensor is about ten times larger than  $r$  of previous sensor. Consequently, the proposed sensor is more practical than previous sensor, because sensitivity of the proposed sensor became higher than that of the previous sensor.

Table 2 shows the density and the viscosity measured by two sensors. It is confirmed that the density and the viscosity could be measured more

Table 1 Eigen frequencies in the air and the liquid derived from analysis in tangential vibration and normal vibration.

Vibration mode	Proposed (Hz)		Previous (Hz)	
	$f_{Air}$	$\Delta f$	$f_{Air}$	$\Delta f$
Tangential	516.3	1.23	136.1	0.71
Normal	499.2	6.04	245.3	33.76

Table 2 Value of liquid density and viscosity under analytic condition and values of that derived from change of resonance frequencies in two vibration modes.

Property	Exact	Proposed	Previous
Density (kg/m <sup>3</sup> )	881	871	897
Viscosity(mPa·s)	270	267	267

Table 3 Comparison of equivalent mass in tangential and normal vibrations of proposed and previous sensor.

Property	Proposed (g)	Previous (g)
$m_{te}$	0.62	0.50
$m_{ne}$	1.04	1.60

accuracy by using the proposed sensor compared to the previous sensor.

### 4. Conclusions

In this paper, we proposed the use of rhombus shape of resonator for liquid density and viscosity sensor. We compared the sensitivity of the proposed sensor to the previous sensor, and found that the sensitivity of the proposed sensor was ten times higher than that of the previous sensor. The obtained results suggested that the measurement range of the density and the viscosity could be increase by using this type of resonator. One of our future plans is to examine range of density and viscosity.

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