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Proceedings of Symposium on Ultrasonic Electronics, Vol. 33 (2012) pp. 115-116
13-15 November, 2012

Examination on Absorbing Boundary Condition Using Method of Characteristics on Collocated Orthogonal Grid

直交格子を用いた特性曲線法における吸収境界に関する検討

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1. Introduction

Time domain numerical analysis for sound wave propagation has been investigated widely as a result of computer development. Now, the development of efficient numerical schemes is an important technical issue.

Method of characteristics (MOC) is used as a time domain numerical analysis method; for example, the CIP (constrained interpolation profile) method, LAX method, and the QUICKEST method.

In one-dimensional analysis, MOCs automatically satisfies no reflection outer boundary[1-2]. However, it is known that these don't satisfy same boundary condition in multi-dimensional analysis. Therefore, efficient absorbing boundary condition for outer boundary to realize accurate MOCs simulations of wave propagation. To overcome this problem, we introduce PML[3-5] (perfectly matched layer) technique, an artificial absorbing layer method for wave equations, for MOCs simulations of wave propagation.

The purpose of this study is introduction and evaluation of PML accurate and we examine the feasibility of PML implementation for 2-D sound field numerical analysis.

2. MOCs simulation of wave propagation

Below, we show the outline of the CIP analysis. The governing equations for linear acoustic fields are

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\nabla p\,, \qquad \nabla\cdot\mathbf{v} = -\frac{1}{K}\frac{\partial p}{\partial t} \qquad (1)$$

In these equations, ρ denotes the density of the medium, K is the bulk modulus, p is the sound pressure, and $\mathbf{v}$ is the particle velocity. Here, we assume that the calculation is for a lossless medium.

In CIP analysis, these equations are transformed into advection forms:

$$\frac{\partial}{\partial t}f_{x\pm} \pm c\frac{\partial}{\partial x}f_{x\pm} = 0, \quad f_{x\pm} = p \pm Zv_x\,. \qquad (2)$$

This transformation process is for the calculation in the x-direction advection. In those equations, Z indicates the characteristic impedance (i.e., $Z=\sqrt{\rho K}$) and c represents the sound velocity in a medium (i.e., $c=\sqrt{K/\rho}$).

Thus, eq. (2) are advection equations of $f_{x\pm}(=p\pm Zv_x)$. We can calculate sound wave propagation by applying the Lagrange interpolation method to these equations. Moreover, considering the advection of $p\pm Zv_y$, we can calculate the propagation in the y-direction as well as in the x-direction.

3. Absorbing boundary in MOCs simulation

While the MOCs simulation automatically satisfies no reflection outer boundary for one-dimensional analysis, automatic outer boundary doesn't have high-efficient absorbing performance for multi-dimensional analysis (See Fig.1).

We introduce PML algorithm technique for MOCs simulations of wave propagation. The PML implemented artificial absorbing constant. In PML, following calculation is additionally required:

$$\frac{\partial}{\partial t}f^{n+1}_{x\pm} \leftarrow -\alpha_x f^{n*}_{x\pm}$$

$$\frac{\partial}{\partial t}f^{n+1}_{y\pm} \leftarrow -\alpha_y f^{n*}_{y\pm}$$

where $f_{x\pm}^{n+1}$ is the value after the calculation of Eq. (2). α_x is an artificial absorbing constant.

4. Result

Figures 1 and 2 show 2-D distribution of sound field at some time steps. Figure 1 is the results without PML, i.e. automatically absorbing boundary. Figure 2 is that with PML. Calculation parameters are the following: the direction of acoustic field propagation, $\pm x, y$ (two-dimensional analysis); grid size, $\Delta x_s = \Delta y_s = 0.05$ m; time step, $\Delta t = 5.0\times10^{-5}$ s; the number of grids, Nx=Ny=200; ρ=1.21 kg/m^3 and K = 1.42×10^5 Pa. Here, the PML (32-layers) is surrounding 200×200 analysis domain.

This result illustrates that PML can reduce the refraction wave from outer boundary. Especially, the oblique reflection from boundaries becomes smaller.

5. Conclusions

In this study, we examine PML absorbing boundary condition for MOCs simulations of multi-dimensional sound field. Though the numerical results, PML implementation can be effective method when the oblique reflection from boundaries is reduced, whereas it uses much memory.

The present method can apply the other MOCs[6] like CIP method. We must develop the PML for GCIP method[7] in the near future.

References

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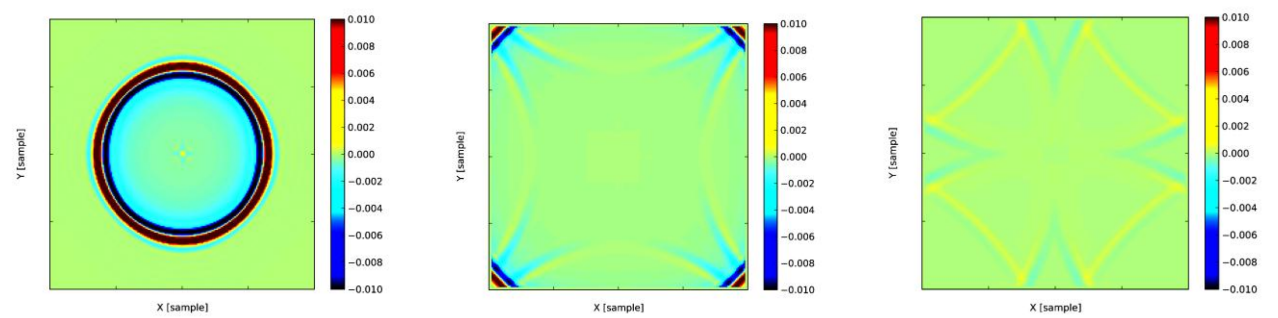
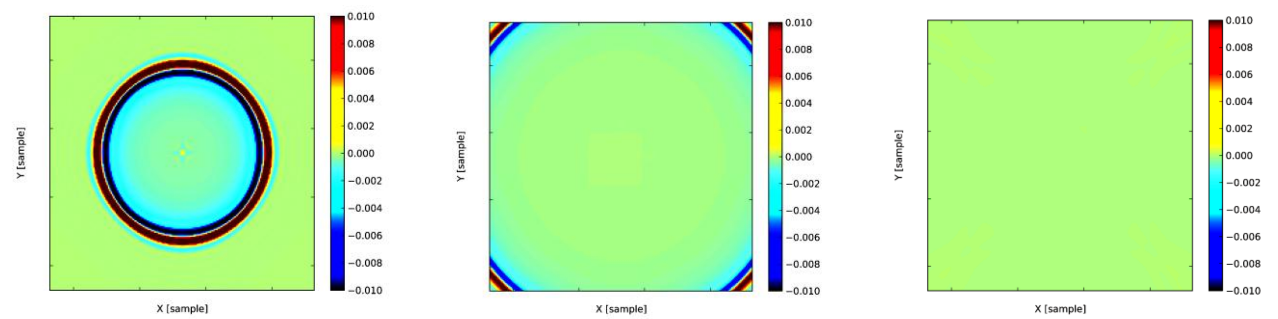


(a) $t = 200\,\Delta t$ (b) $t = 400\,\Delta t$ (c) $t = 600\,\Delta t$.

Figure 1 2-D distribution of sound field(without PML)



(a) $t = 200\,\Delta t$ (b) $t = 400\,\Delta t$ (c) $t = 600\,\Delta t$.

Figure 2 2-D distribution of sound field(with PML)