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Elucidation of Change in Motional Capacitance of Quartz-Crystal Tuning-Fork Tactile Sensor Induced by Viscoelastic materials in Contact with Its Base

基底部に接触した粘弾性体で生じた音叉型水晶触覚センサの動的容量変化の解明

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1. Introduction

If we want to measure both Young's modulus and viscosity coefficient of materials using a conventional instrument, an adequate instrument to fit the hardness type of samples must be chosen. For example, a tension test is used for measuring Young's modulus of hard materials such as plastics or metals. On the other hand, rotary viscometer[1] and dynamic mechanical analysis system[2] (DMA) are also used for measuring Young's modulus and viscosity coefficient of soft materials such as rubbers.

In this study, we prove theoretically and experimentally that the change in the motional capacitance of quartz-crystal tuning-fork tactile sensor varies with both the dynamic Young's modulus and viscosity of amorphous polymers such as neoprene rubbers.

2. Experiment on tactile sensor's motional capacitance in contact with neoprene rubbers and plastics and on DMA in contact with neoprene rubbers

The experimental method is fundamentally the same as that in ref. 3 except a 10 gf load.

We already derived the analytical formula of the motional capacitance C_a, of a quartz-crystal tuning-fork tactile sensor by approximating the right half of the quartz-crystal tuning fork as an L-shaped bar and using Winkler's foundation model as contact material and conservation law of energy that the electrostatic energy in motional capacitance is equal to the sum of the strain energy of the quartz arm and the stored energy of the materials in contact with its base, eq.(5) in ref. 4 as;

$$C_a = \frac{Q^2/2}{\frac{K_c}{4}\int_0^{l_2}\left(\frac{\partial^2 u_2}{\partial x_2^2}\right)^2 dx_2 + \frac{k}{4}\int_0^{l_1} u_1^2 dx_1}, \qquad (1)$$

where Q is the electrical charge, K_c is the bending stiffness of the arm, l_1 is the length of the half of the base, l_2 is the length of the arm, u_1 is the flexural displacement of the base, u_2 is the flexural displacement of the arm, and k is the Winkler's foundation coefficient and is assumed to be equal to the Young's modulus of material.

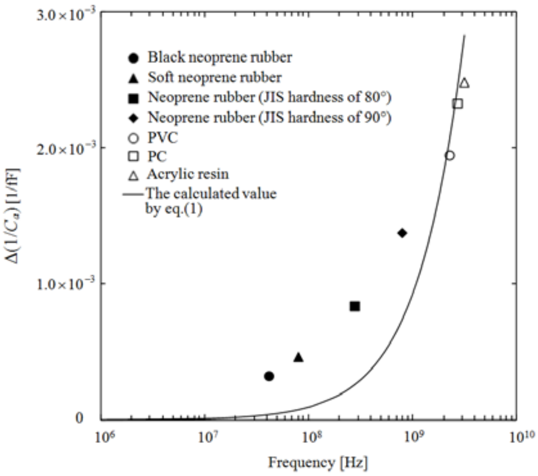

Fig.1. Measured $\Delta(1/C_a)$ vs Young's moduli of materials for neoprene rubbers and plastics.

Figure 1 shows the change in the reciprocal measured motional capacitance $\Delta(1/C_a)$ for neoprene rubbers and plastics against the Young's moduli of materials. The solid line in Fig. 1 is the calculated values of $\Delta(1/C_a)$ using both eq. (1) and the values of the Young's moduli indicated in the horizontal coordinate in Fig. 1. As for plastics in Fig. 1, the theoretical value of $\Delta(1/C_a)$ are closer to the measured values of $\Delta(1/C_a)$ against Young's moduli obtained by a tension test in cooperation with a strain gauge because plastics are elastic materials. However, as for neoprene rubbers, the measured values of $\Delta(1/C_a)$ against Young's moduli using about 32.5 kHz obtained from DMA (UBM Rheogel-E4000) are higher than the calculated values of $\Delta(1/C_a)$.

3. Analysis of tactile sensor's motional capacitance formula based on the viscoelastic foundation

From eqs. (10) and (18) in ref. 4, the equation of motion for the base in contact with viscoelastic materials depicted by the Voigt viscoelastic body and the equation of motion for the arm are given by

$$E_1 I_1 \frac{\partial^4 u_1}{\partial x_1^4} + P_t \frac{\partial^2 u_1}{\partial x_1^2} + A_1 \rho \frac{\partial^2 u_1}{\partial t^2} + \mu \frac{\partial u_1}{\partial t} + k u_1 = 0, \quad (2)$$

$$K_c \frac{\partial^4 u_2}{\partial x_2^4} + A_2 \rho \frac{\partial^2 u_2}{\partial t^2} = 0, \quad (3)$$

where E_1 is the Young's modulus of the base, I_1 is the moment of inertia of the base, P_t is the lateral clamping force, A_1 is the cross-sectional area of the beam, A_2 is the cross-sectional area of the arm, ρ is the mass density of quartz crystal, μ is the viscosity coefficient of viscoelastic materials.

Since the solution of eq. (2) is generally complex one, it is not convenient for the later calculation. Therefore, we should adopt the analytical method not to yield the complex solution. After the tedious calculation, we can obtain the following motional capacitance formula including the viscosity of viscoelastic materials as

$$C_a = \frac{2Q^2}{K_c \int_0^{l_2} \left(\frac{\partial^2 u_2}{\partial x_2^2} \right)^2 dx_2 + k \int_0^{l_1} u_1^2 dx_1 + 4P_4} \quad (4)$$

The loss energy P_4 for the viscosity term in eq. (2) is given by

$$P_4 = E'' \int_0^{l_1} \left(\int_0^{\frac{2\pi}{\omega_n}} \left(\frac{\partial}{\partial t} (z_n \cdot \phi_{n1}) \right)^2 dt \Bigg/ \left(\frac{2\pi}{\omega_n} \right) \right) dx_1, \quad (5)$$

where E'' is the loss modulus.

4. Experiments on tactile sensor's motional capacitance in contact with neoprene rubbers and Discussion

The values, subtracting the calculated reciprocal of eq. (1) by use of the values of dynamic Young's moduli of neoprene rubbers at 32.5 kHz obtained using DMA from the measured values of $\Delta(1/C_a)$ are indicated by closed marks with respect to dynamic viscosity of neoprene rubbers at 32.5 kHz in Fig. 2. The values, subtracting the calculated reciprocal of eq. (1) by use of the value of dynamic Young's modulus of neoprene rubbers at 32.5 kHz from the calculated reciprocal of eq. (4) by use of

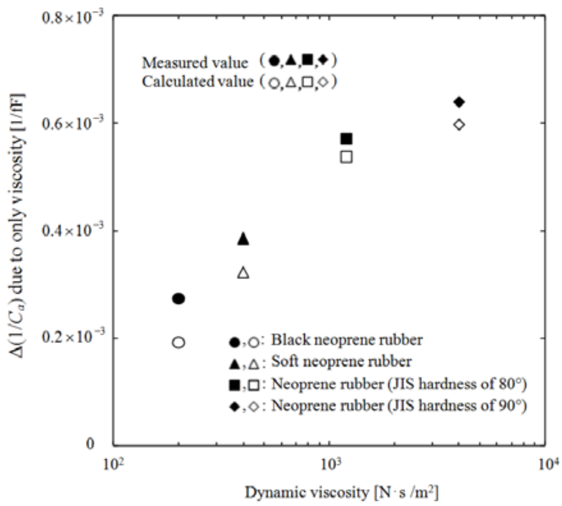

Fig.2. $\Delta(1/C_a)$ due to only viscosity vs dynamic viscosity of neoprene rubbers.

the values of dynamic viscosity of neoprene rubbers at 32.5 kHz, are also indicated by open marks at the same time in Fig. 2. These are shown by $\Delta(1/C_a)$ due to only viscosity. From Fig.2, we may conclude that the change in the motional capacitance of quartz-crystal tuning-fork tactile sensor changes with both dynamic Young's modulus and viscosity of viscoelastic materials at 32.5 kHz.

5. Conclusions

We derived the analytical formula of the motional capacitance of the quartz-crystal tuning-fork tactile sensor based on the combination of L-shaped bar's, viscoelastic foundation models, and conservation low of energy between the electrostatic energy in motional capacitance and the sum of the strain energy of the quartz arm and the stored and loss energies of the materials in contact with its base. We found that the change in reciprocal motional capacitance, before and after the sensor's base getting into contact with materials, was quantitatively yielded by the effect of both their dynamic Young's modulus and viscosity of neoprene rubbers at 32.5 kHz.

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