

Sub Grid Technique for CIP-MOC Simulation of Sound Wave Propagation

サブグリッドを用いた CIP-MOC 法による音波伝搬シミュレーション

Yuta Ara^{1†}, Kan Okubo¹, Norio Tagawa¹ and Takao Tsuchiya² (¹Facult. System Design, Tokyo Met. Univ.; ²Facult. Sci. Eng., Doshisha Univ.)

荒 佑多^{1‡}, 大久保寛¹, 田川憲男¹, 土屋隆生² (¹首都大 システムデザイン; ²同志社大 理工)

1. Introduction

To date, numerical analysis for sound wave propagation in time domain has been investigated widely as a result of computer development[1-2]. Now, the development of accurate numerical schemes is an important technical issue.

The Constrained Interpolation Profile (CIP) method is a novel low-dispersive numerical scheme[3-8]. It is a kind of method of characteristics (MOC)[9]. In our past study, we have applied the CIP method to numerical analyses of sound wave propagation.

The feature of the CIP method is that it uses the values of acoustic field and their spatial derivatives at grid points to solve the problem of wave propagation. The family of this scheme is called "Multi-Moment Scheme".

In the meanwhile, sub grid techniques[10-11] are proposed for simulations of complicated heterogeneous media or large-scale simulations of wave propagation. This method has an advantage regarding small memory used. We can make a cell size smaller the surrounding cells for complicated modeling.

In this study, we propose sub grid techniques for CIP-MOC simulation of sound wave propagation.

2. Acoustic simulation using CIP method

The governing equations for linear acoustic fields are given in Eq. (1) and Eq. (2):

$$\rho \frac{\partial \vec{v}}{\partial t} = -\nabla p, \quad (1)$$

$$\nabla \cdot \vec{v} = -\frac{1}{K} \frac{\partial p}{\partial t}. \quad (2)$$

In those equations, ρ denotes the density of the medium, K is the bulk modulus p is sound

pressure and v is the particle velocity. Here we assume that the calculation is for a lossless medium. Moreover, assuming $\vec{v} = (v_x, 0, 0)$ in order to analyze one-dimensional (1-D) acoustic field propagation in the x -direction, we can obtain the following equations from Eq. (1) and Eq. (2).

$$\frac{\partial v_x}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \quad (3)$$

$$\frac{\partial p}{\partial t} + K \frac{\partial v_x}{\partial x} = 0 \quad (4)$$

Then, by addition and subtraction of these two equations, we obtain

$$\frac{\partial(p \pm Zv_x)}{\partial t} \pm c \frac{\partial(p \pm Zv_x)}{\partial x} = 0 \quad (5)$$

In those equations, Z indicates the characteristic impedance (i.e. $Z = \sqrt{\rho K}$) and c represents the sound velocity in medium (i.e. $c = \sqrt{K/\rho}$).

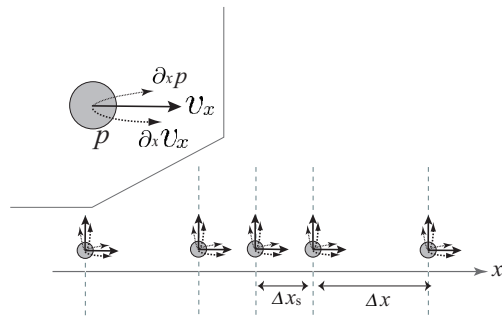
In addition, through simple spatial differentiation of the equations, the equations of the derivatives are given as

$$\frac{\partial(\partial_x p \pm Z \partial_x v_x)}{\partial t} \pm c \frac{\partial(\partial_x p \pm Z \partial_x v_x)}{\partial x} = 0 \quad (6)$$

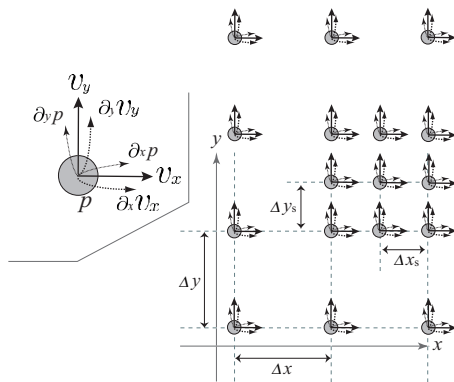
Then, Eqs. (5) and (6) are advection equations of $p \pm Zv_x$ and $\partial_x p \pm Z \partial_x v_x$. We can calculate sound wave propagation by applying the CIP method to these equations. Moreover, considering advection of $p \pm Zv_y$ and $\partial_y p \pm Z \partial_y v_y$, we can calculate the propagation in the y -direction as well as in the x -direction.

3. Sub grid techniques in CIP method

Figure 1 shows the aspect of sub grid technique in CIP method; (a) one dimensional model and (b) two dimensional model. Here, Δx and Δx_s represent the course grid size and sub grid size, respectively.



(a) one dimensional model



(b) two dimensional model

Fig. 1 Sub grids in CIP method

4. Results and discussion

We show the numerical results obtained using the sub grid technique. Calculation parameters are: the direction of acoustic field propagation, $\pm x$ (1D analysis); grid size, $\Delta x = 0.06$ m; sub grid size, $\Delta x_s = 0.02$ m.

Figure 2 shows the sound pressure distribution obtained using sub grid CIP analysis at $t = 400 \Delta t$, $t = 460 \Delta t$ and $t = 520 \Delta t$ respectively, where $\rho = 1.21$ kg/m³ and $K = 1.4236 \times 10^5$ Pa. Here, the sub grids are assumed in the region of $28.8 \leq x \leq 29.6$. We can confirm the correct propagation behavior including the sub grid region.

Sub grid technique in the CIP analysis just needs to change interpolating function in sub grid region, because CIP scheme is based on a two-point stencil. Therefore, it can be implemented without additional more complicated programming.

5. Conclusion

This study examines sub grid techniques for CIP-MOC simulation of sound wave propagation.

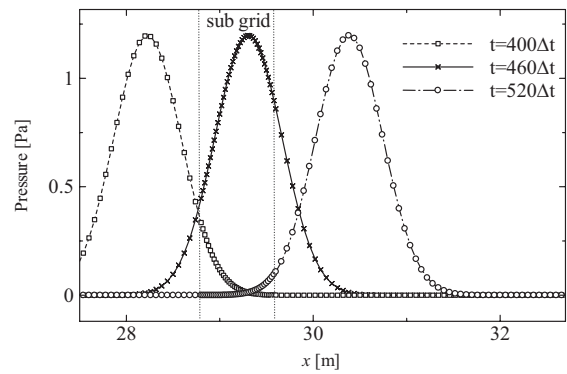


Fig. 2 Distribution of the sound pressure at $t = 400 \Delta t$, $t = 460 \Delta t$ and $t = 520 \Delta t$.

Though numerical results, we can evaluate the sub grid techniques in CIP analysis. The calculation using more complicated analysis model is a future work.

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