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Low-Frequency Forbidden Bands in Phononic Crystal Plates with Helmholtz Resonators

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1. Introduction

Helmholtz resonator (HR) has been used to low-frequency sound and noise absorption for a long time because of its resonance characteristics.[1] A typical HR contains a cavernous space (cavity), in which there is a small tube (neck) estabilising a transmission between the interior and exterior gas. The small tube opening results in long-wavelength vibrations.[2] An HR array can be a small impdeance mismatch composite with air for airborne sound and used to build a convergent lens with good focusing effect.[3] Moreover, sound energy harvesting utilizing an HR with a piezoelectric backplate had also been demonstrated.[4] We note that, up to now, attenuation in a solid-type HR array has not be studied.

In this paper, propagation of elastic wave in a thin plate with square-array HR is investigated. It is shown that the resonators can open up not only the Bragg gaps but also low-frequency resonance gaps, and the band-gap frequencies can be tuned by the geometric sizes of the neck and cavity of the HR.

2. Model and Method of Calculation

As shown in **Fig. 1**, the Helmholtz resonators consist of two cylinder segments (i.e., the neck and the cavity) with different lengths and radii. Their geometrical parameters are labeled as follows: the length and radius of neck are l and r, respectively, and those of cavity are L and R, respectively. The considered phononic structure is composed of the Helmholtz resonators grafted periodically on a thin plate of thickness h, and the resonators are arranged in square lattice with a lattice spacing a. To study the acoustic forbidden bands in the structure, the calculations of dispersion relations and transmission spectra are conducted. The dispersion relations are calculated for the infinite system utilizing a finite element (FE) method, in which only the unit cell (Fig. 1) is meshed, and the Bloch periodic boundary conditions (PBCs)[5] are implemented based on the Bloch theorem. The PBCs define a wave number related phase relation on the boundary between adjacent cells and are respectively applied on the four lateral faces of the unit cell. By varying the wave vector in the first Brillouin zone and solving the eigenvalue problem, the dispersion relations and eigenmodes are obtained.

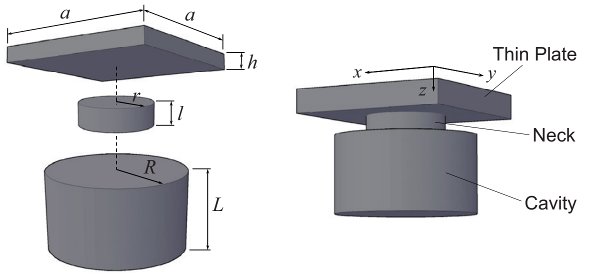

Fig. 1 Schematics of the unit cell of the PC plate with a solid Holmholtz resonator.

To calculate the transmission spectra, a finite structure containing N layers of resonators equally spaced out with the distance a along the x-direction on the thin plate is considered. Elastic plane waves with single frequency are continuously excited by an associated monotonic line source and impinge to the structure along the x-direction. To prevent wave reflections from the domain boundaries, the perfect matching layers (PMLs) attenuating wave energy are implemented as extended domains. By varying the excitation frequency of the line source, energy spectra of transmission through the finite structure can be obtained.[5]

3. Results and Discussion

Figure 2 shows the dispersion relations of the Helmholtz resonator phononic crystal plate (HRPC plate). The whole structure is assumed to be made of steel. The mass density, Young's modulus, and Poisson's ratio are set to be 7780 kg/m^3, 226 GPa, and 0.29, respectively. In the calculation of Fig. 2, a=10 mm, h=1 mm, r=3 mm, l=1 mm, R=4.5 mm, and L=4 mm are assumed, respectively.

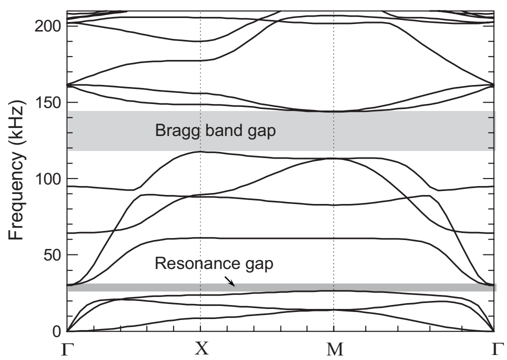

Fig. 2 Dispersion curves with two complete band gaps of elastic waves in the HRPC plate.

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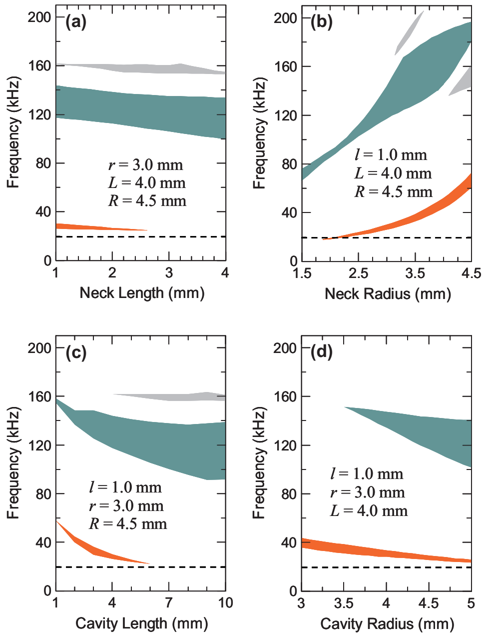

Fig. 3 Gap maps as a function of the HR geometrical parameters. Band gap variations with the changes of the cavity radius R (a), cavity length L (b), neck radius r (c), and neck length l (d), respectively.

In Fig. 2, two complete band gaps are found. The lower one is about 26–30 kHz and the upper one 118–144 kHz. It can be shown that the lower complete band gap results from the resonance of the resonators, while the upper gap is formed mainly by the Bragg diffraction. By the resonance, the lower gap frequencies are as low as about one-fifth of the higher gap frequencies. Note that when the size of the unit cell is doubled or larger, the frequencies of the lower gap can be down to the audible regime, in which the sound induced vibrations can be blocked with the proposed HRPC solid structure.

Figure 3 shows the gap maps as a function of the geometric factors R, L, r, and l, of the resonators, respectively. It can be observed that the resonance band gap width and frequencies are much more significantly related to these factors than the Bragg gap. Recall the well-known equation evaluating the lowest resonance eigenfrequency f_r of an HR for fluid:

$$f_r \approx \frac{c}{2\pi}\sqrt{\frac{r^2}{R^2 L l}}, \tag{1}$$

where c is the sound speed of the fluid. The value of the resonance gap frequencies follow similar trends predicted by Eq. (1), but it can not be worked out with the formula since the neck has to be small and dispersive wave velocities in the solid resonator has to be considered. From Fig. 3, the neck radius r is the most decisive factor for the resonance (lower) gap frequency variation. In Fig. 3(c), the mid-gap frequency of the resonance gap has 380% change at most. It can be reduced to 20 kHz when r=1.75mm.

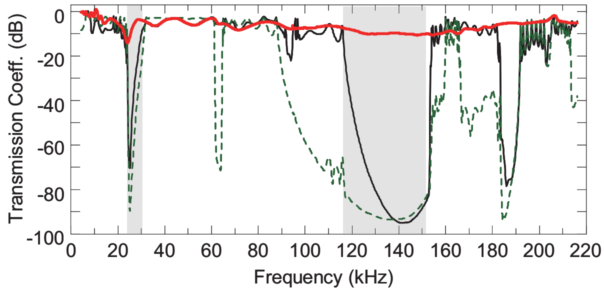

Fig. 4 Transmission spectra of elastic wave through an eight-cell and a single cell finite HRPC plate structures.

The transmission spectra for elastic waves propagating through an 8-layered HRPC (i.e., N=8) are shown in **Fig. 4**. Solid curve denotes that the x-, y-, z-polarized line sources are given simultaneously, and dashed curve denotes the y-polarized source is applied only. It is observed that over 60 dB energy attenuation occurs within the band gap frequencies predicted by the dispersion relations of the infinite system. Attenuation in the partial band gap in the ΓX direction is also observed. Moreover, the case of the y-polarized line source exhibits more and wider gaps, which correspond to the deaf bands due to the ineffective excitation of the associated eigenmodes in the HRPC plate. The transmission for the case of single HR (i.e., N=1) is also plotted in Fig. 4 (bold solid curve). A dip is found near the bottom edge of the resonance gap which reveals the fact that energy at the resonance frequency is considerably reflected by the HR due to resonances. As a result, the square HR array creates the low-frequency resonance gap.

4. Conclusion

In this paper, low-frequency forbidden bands of elastic waves in a square HR array on a thin plate are presented. The resonance characteristics to open the band gaps are demonstrated. The neck radius is recognized as the most decisive factor to vary the resonant gap frequency. Transmission spectra show evident results of resonance and attenuation within the band gaps of finite HRPC systems.

Acknowledgment

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