

## Shear wave velocity estimation by virtual sensing array spectrum analysis 仮想センシングアレイスペクトル解析による生体内すり弾性波速度推定

Takashi Miwa and Yoshiki Yamakoshi (Grad. School of Eng., Gunma Univ.)  
三輪 空司<sup>†</sup>, 山越 芳樹 (群馬大 院工)

### 1. Introduction

It is expected that shear wave velocity is a quantitative parameter in tissue elasticity measurement. However, the resolution and the accuracy of the measurement depend on both the frequency of the shear wave and its propagation characteristics inside the ROI. As a medical diagnosis requirement, the estimation error and spatial resolution are expected to be within 10 % and 5 mm, respectively, for breast tissue. But those are not necessarily achieved in the conventional systems because multiple reflection and complex 3D propagation of the shear wave in the soft tissue greatly affect on the accuracy of the velocity estimation.

Multi-points measurement of tissue displacement in the direction of the ultrasonic beam propagation is equivalent to 1D Virtual Sensing Array, which we call hereafter "VSA". The simultaneous displacement measurement using multiple ultrasonic transducers gives complete set of data acquired by 2D or 3D VSA, which is one of the solutions of the above problem. However, as the frequency of the shear wave increases for improving spatial resolution, the estimation error of the displacement distribution of the shear wave increases in the conventional arc-tangent method.<sup>1)</sup>

In this paper, a novel shear wave velocity estimation method is proposed by using a deconvolution-based displacement estimation method and by a VSA spectrum analysis. The minimum aperture of VSA, in which 10% estimation error is achieved, is discussed through numerical simulations.

### 2. Displacement Estimation with VSA

Basic idea of this method is that a wave number vector of a shear wave is estimated by spectrum analysis of complex displacement obtained with Virtual Sensing Array (VSA). **Figure 1** shows the concept of VSA velocity estimation. A shear wave source placed at the origin vibrates with a low frequency of  $f$ . The tissue displacement  $\xi_z(t, \mathbf{p})$  in z-component is expressed as

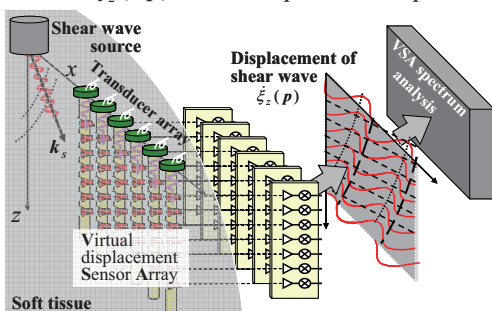


Fig.1 Concept of shear wave velocity estimation using virtual sensing array spectrum analysis.

$$\xi_z(t, \mathbf{p}) = \delta \cos(2\pi ft - \mathbf{k}_s \cdot \mathbf{p}) \quad (1)$$

where  $\delta$ ,  $\mathbf{p}$ , and  $\mathbf{k}_s$  are displacement amplitude of the shear wave, a position vector  $(x, z)$ , and wave number vector of the shear wave to be estimated, respectively. Assuming that the ultrasonic wave propagates only to the  $z$  direction with no attenuation for simplicity, complex Doppler signals are obtained by convolution between a point spread function (PSF)  $w(\mathbf{p})$  and a US-scatterer-reflection-coefficient distribution  $\dot{\gamma}(\mathbf{p})$  modulated by vibration propagation with the wave number of  $\mathbf{k}_s$  as

$$\dot{g}(\mathbf{p}, t) = \iint w(\mathbf{p} - \mathbf{p}') \dot{\gamma}(\mathbf{p}') \exp\{-2jk_u \xi_z(t, \mathbf{p}')\} dx' dz' \quad (2)$$

where  $k_u$  is the wave number of the ultrasonic wave. If  $k_u \delta$  is much smaller than the unity, eq.(2) can be represented with  $\angle \exp(j\theta) \cong 1 + j\theta$  as

$$\dot{g}(\mathbf{p}, t) \cong \iint w(\mathbf{p} - \mathbf{p}') \dot{\gamma}(\mathbf{p}') \{-2jk_u \xi_z(t, \mathbf{p}')\} dx' dz' \quad (3)$$

By applying Fourier transformation for time  $t$  and taking the frequency components at 0 and  $f$ , we have

$$\dot{g}_0(\mathbf{p}) = \iint w(\mathbf{p} - \mathbf{p}') \dot{\gamma}(\mathbf{p}') dx' dz' \quad \text{and} \quad (4)$$

$$\dot{g}_f(\mathbf{p}) = -jk_u \delta \iint w(\mathbf{p} - \mathbf{p}') \dot{\gamma}(\mathbf{p}') \exp(j\mathbf{k}_s \cdot \mathbf{p}') dx' dz' \quad (5)$$

When the wavelength  $\lambda_s$  of the shear wave is much larger than the effective area of the PSF,  $\exp(j\mathbf{k}_s \cdot \mathbf{p}')$  can be approximately extracted outside the integration. Thus, the phase difference of eq.(3) is approximated by  $\dot{g}_f(\mathbf{p})/\dot{g}_0(\mathbf{p})$  which is the displacement distribution estimated by an arc-tangent method. However, as  $\lambda_s$  decreases, the above approximation is not satisfied. Since, generally,  $\dot{g}_0(\mathbf{p})$  has many null points caused by interference of the multiple scatterers in the PSF, the error of the displacement estimation increases due to division by small amplitude.

### 3. Deconvolution-based Velocity Estimation

In order to estimate the wave number  $\mathbf{k}_s$  more accurately, we have to eliminate  $w(\mathbf{p})$  and  $\dot{\gamma}(\mathbf{p})$  from Eq.(5). If the PSF is known, the deconvolution filter  $H(\mathbf{k}, a)$  like a Wiener filter is applicable.

$$H(\mathbf{k}, a) = |W(\mathbf{k})|^{2a-1} \exp(-j\angle W(\mathbf{k})) \quad (6)$$

where  $W(\mathbf{k})$  is 2D Fourier transformation(FT) of  $w(\mathbf{p})$ . When  $a$  is 1 and 0,  $H(\mathbf{k}, a)$  acts as the matched and inverse filter, respectively. Thus, we can estimate a pseudo-reflection coefficient distribution of scatterers as

$$\dot{\gamma}_{est}(\mathbf{p}, a) = \mathfrak{I}^{-1}[G_0(\mathbf{k})H(\mathbf{k}, a)] = \dot{\gamma}(\mathbf{p}) \otimes \mathfrak{I}^{-1}[|W(\mathbf{k})|^{2a}] \quad (7)$$

where  $\mathfrak{I}$  and  $G_0(\mathbf{k})$  mean the 2D-FT operator and 2D-FT of  $\dot{g}_0(\mathbf{p})$ , respectively. Also by applying this filter to  $\dot{g}_f(\mathbf{p})$ , the following function including the shear wave propagation term is also obtained as

$$\dot{\gamma}_{\text{mod}}(\mathbf{p}, a) = -jk_u \delta \exp(j\mathbf{k}_s \cdot \mathbf{p}) \dot{\gamma}_{\text{est}}(\mathbf{p}, a) \quad (8)$$

Here, if we can eliminate  $\dot{\gamma}_{\text{est}}(\mathbf{p}, a)$  from  $\dot{\gamma}_{\text{mod}}(\mathbf{p}, a)$ , the propagation term can be extracted with the same manner like Wiener filter as

$$\eta(\mathbf{p}, a, b) = |\dot{\gamma}_{\text{est}}(\mathbf{p}, a)|^{2b-1} \exp\{-j\angle \dot{\gamma}_{\text{est}}(\mathbf{p}, a)\}. \quad (9)$$

So, the wave number spectrum is given by

$$I(\mathbf{k}) = \mathfrak{F}[j\dot{\gamma}_{\text{mod}}(\mathbf{p}, a)\eta(\mathbf{p}, a, b)] = k_u \delta \Gamma_{\text{acf}}(\mathbf{k} - \mathbf{k}_s, a, b) \quad (10)$$

Since  $\Gamma_{\text{acf}}(\mathbf{k}, a, b)$  is given by 2D-FT of  $|\dot{\gamma}_{\text{est}}(\mathbf{p}, a)|^{2b}$   $\Gamma_{\text{acf}}(\mathbf{k}, a, b)$  always has a main peak at  $(k_x, k_z) = (0, 0)$ . Therefore, the peak position  $\mathbf{k}_{\text{est}}$  of  $I(\mathbf{k})$  gives the more accurate wave number of the shear wave. The mean shear wave velocity in the VSA is evaluated by

$$v_{\text{est}} = 2\pi f / |\mathbf{k}_{\text{est}}| \quad (11)$$

#### 4. Simulation model

The spatial resolution and velocity estimation accuracy are evaluated by numerical simulations. This simulation is based on scattering from numerous point scatterers for the ultrasonic wave, which can fluctuate by shear wave propagation. The number of scatterers is more than 50,000. Attenuation of the shear wave is neglected. An effect of multiple shear waves is taken into account. The ultrasonic pulse has the center frequency  $f_u$  of 5 MHz and the burst length of  $4\lambda_u$ . The ultrasonic beam is weighted with a cosine function having the width of 5 mm.

In order to statistically evaluate the bias error and standard deviation of the estimated velocity, 40 different scatterer structure distributions are employed. We define the minimum size of the square VSA aperture as ‘‘Minimum aperture’’ which satisfies that the error of estimation averaged for three measurements is within  $\pm 10\%$  as follows

$$\sqrt{3}E_\sigma(w) + |E_b(w)| < 10\% \quad (12)$$

where  $w$ ,  $E_b(w)$  and  $E_\sigma(w)$  are VSA aperture, bias error and standard deviation of estimation.

#### 4. Simulation results

**Figure 2** shows estimated  $E_b(w)$  and  $E_\sigma(w)$  using the proposed method in the wavelength  $\lambda_s$  of 5 mm. Fig.2(a) discusses the case when only a direct wave exists in a ROI.  $a$  and  $b$  in eq.(9) are optimized as 0.5 and 0.6. The bias error is negligible in the aperture more than 2 mm. As the VSA aperture decreases,  $E_\sigma(w)$  increases. The results using the arc-tangent method are also plotted. It is found that the proposed method has significantly smaller  $E_\sigma(w)$  than the arc-tangent method in the narrow VSA aperture. So, the proposed method is effective to improving the precision of the velocity estimation.

Fig.2(b) discusses the case when both a direct and reflected wave exist in a ROI as a function of the relative direction of the reflected wave against the direct wave. The reflection coefficient of the reflected wave is 0.3.  $E_b(w)$  significantly depends on the VSA aperture and the relative direction. The increase of the bias error is

caused by overlapping the spectrum of the main-lobe for the reflected wave with that for the direct wave. Thus, existence of reflected waves lowers the accuracy of the velocity estimation.

**Figure 3(a)-(d)** show the minimum aperture as a function of the relative direction and reflection coefficient of the reflected wave in the wavelength of 2, 3, 4, and 5 mm, respectively. In Fig.3, the minimum aperture is a function of not only the relative direction but also the reflection coefficient, which increases the minimum aperture due to overlap of the spectrum. As the wavelength increases, the minimum aperture also increases because the peak of direct and reflected wave approaches to the origin in wave number spectrum.

#### 5. Conclusions

We propose a shear wave velocity estimation method using spectrum analysis from the virtual sensing array data. The accuracy and the VSA aperture is discussed through numerical simulations. When the reflection coefficient is less than 0.3, the minimum aperture of 3.5 mm can be achieved within the 10% accuracy in whole measurement area even if the wavelength of the shear wave changes from 2 to 5 mm due to inhomogeneity of the propagation velocity.

#### References

1. Y. Yamakoshi, et.al.: IEEE Trans. UFFC, **37** (1990) 45.

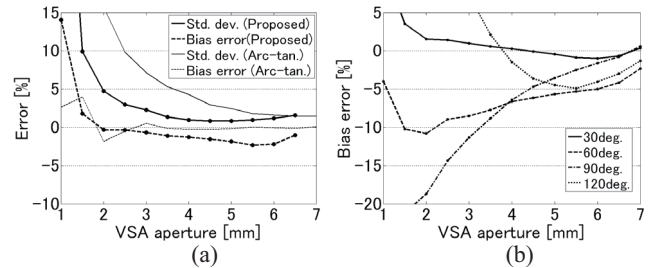


Fig.2 Estimation error of shear wave velocity as a function of VSA aperture by using proposed method ( $\lambda_s = 5$  mm). We simulate the case when (a): only a direct wave and (b) both a direct and reflected waves exist in the ROI.

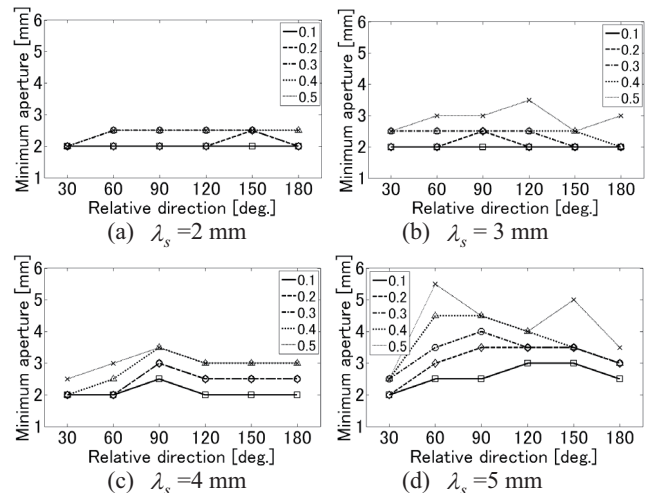


Fig.3 Estimated minimum aperture as a function of relative direction and reflection coefficient of the reflected wave in the wavelength of 2, 3, 4, and 5 mm, respectively. The legend of the figure shows the reflection coefficient.