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High Accuracy Measurement of Ultrasonic Travel Time by Pulse-Echo Overlap Method: the McSkimin Criterion for Specimens over a Wide Range of Characteristic Impedance

パルスエコー重畳法による超音波伝播時間の高確度測定:広域特性インピーダンス試料への McSkimin 判定条件

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1. Introduction

When measuring ultrasonic travel time in solids with the pulse-superposition method[1,2] or the pulse-echo overlap method,[3,4] the McSkimin criterion is often used to find the condition of correct overlapping between echoes.[1,2] By making use of phenyl benzoate, the transducer can be bonded to the specimen as thin as possible[4]; also, the characteristic impedance of phenyl benzoate has been evaluated[5]. Thus, the McSkimin criterion is calculated for phenyl benzoate as the adhesive and for specimens over a wide range of characteristic impedance. As a result, the number of mismatch cycles in the overlapping is clearly determined.

2. The McSkimin criterion

First, we shall summarize the theory[1,2] briefly. From the acoustic transmission line analysis of the composite structure shown in **Fig. 1**, the mechanical impedance, Z_d, of the transducer, including the bonding layer, seen from FF' is given by

$$Z_\mathrm{d} = \mathrm{j}Z_1\left[\frac{(Z_1/Z_2)\tan\beta_1 l_1 + \tan\beta_2 l_2}{(Z_1/Z_2) - \tan\beta_2 l_2 \tan\beta_1 l_1}\right], \quad (1)$$

and the reflection coefficient E_b/E_i for sound pressure wave is given by

$$E_\mathrm{b}/E_\mathrm{i} = (Z_\mathrm{d} - Z_\mathrm{s})/(Z_\mathrm{d} + Z_\mathrm{s}), \quad (2)$$

where j is the imaginary unit; Z_1, Z_2, and Z_s are the characteristic impedances of the bonding material, the transducer, and the specimen, respectively; β_1 and β_2 are the phase constants for the bonding material and the transducer, respectively; l_1 and l_2 are the thicknesses of the bonding layer and the transducer, respectively; E_i and E_b are the complex amplitudes of incident and reflected sound pressure waves, respectively. Then, the phase angle, γ, of the reflection coefficient is given by

$$\gamma = -2\arctan(Z_\mathrm{d}/\mathrm{j}Z_\mathrm{s}) \quad (3)$$

The travel time is usually measured between adjacent echoes. The time relation under the cycle-for-cycle overlap between adjacent echoes is

$$T = \tau - \gamma/(360\nu) + n/\nu, \quad (4)$$

where T is the measured travel time, τ is the true round-trip travel time, ν is the radio frequency of the pulse, and n is the number of mismatch cycles in the overlap.

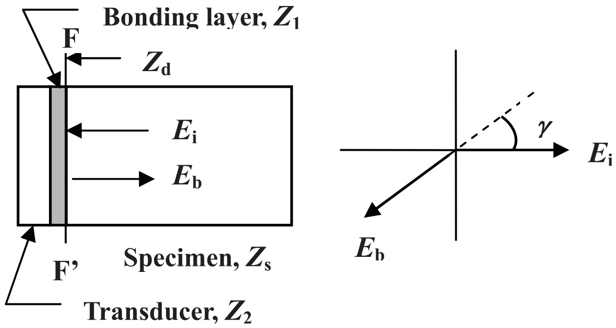

Fig. 1 Composite structure of transducer, bonding layer, and specimen.

In order to find the condition of correct cycle-for-cycle matching, n = 0, the McSkimin criterion uses two different frequencies, ν_r the resonance frequency of the transducer and $0.9\nu_r$. Let $T(\nu_r)$ and $T(0.9\nu_r)$ be the measured travel times at ν_r and $0.9\nu_r$ and let $\gamma(\nu_r)$ and $\gamma(0.9\nu_r)$ be the phase angles at ν_r and $0.9\nu_r$, then

$$\Delta T = T(0.9\nu_\mathrm{r}) - T(\nu_\mathrm{r})$$
$$= \frac{1}{0.9\nu_\mathrm{r}}\left[n - \frac{\gamma(0.9\nu_\mathrm{r})}{360}\right] - \frac{1}{\nu_\mathrm{r}}\left[n - \frac{\gamma(\nu_\mathrm{r})}{360}\right]. \quad (5)$$

If Z_1 and Z_2 are known, $\gamma(\nu_r)$ and $\gamma(0.9\nu_r)$ can be evaluated for given values of $\beta_1 l_1$ and Z_s, and hence ΔT at predetermined n can be calculated.

3. Cycle-for-cycle matching between echoes

From eqs. (1)-(5), ΔT is calculated for 10 MHz quartz transducers (Z_2 = 15.3×10^6 kg m^{-2} s^{-1} for X-cut quartz and 10.4×10^6 kg m^{-2} s^{-1} for Y-cut quartz[2]) since the properties of quartz transducers are accurately known. Phenyl benzoate was chosen as the bonding material since it can be easily obtained as a pure chemical and is known to have superior qualities near room temperature[4]; also its characteristic impedance has been evaluated (Z_1 = 3.30×10^6 kg m^{-2} s^{-1} for the longitudinal wave and 1.60×10^6 kg m^{-2} s^{-1} for the transverse wave).[5] Values of Z_s were taken from 5 to 30×10^6 kg m^{-2} s^{-1} for the longitudinal wave and from 3 to 18×10^6 kg m^{-2} s^{-1} for the transverse wave; these ranges adequately cover ordinary specimens. Careful bonding makes $\beta_1 l_1$ less than 10° at 10MHz, so that ΔT is calculated at $\beta_1 l_1$ = 0°, 10°, and 20°.

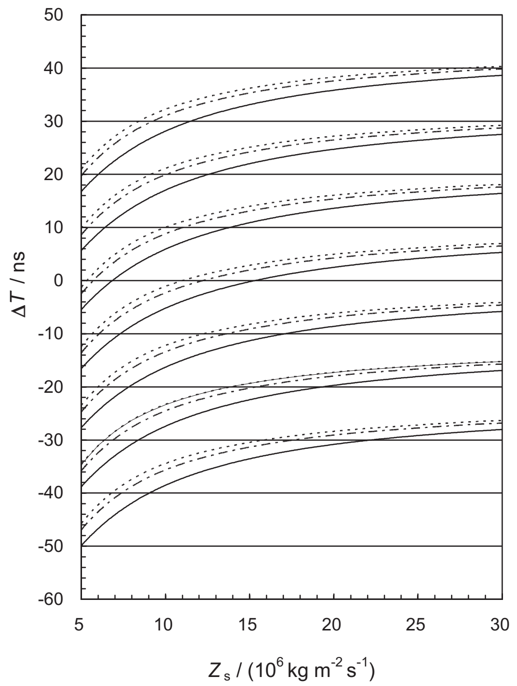

Fig. 2 ΔT as a function of Z_s, n, and $\beta_1 l_1$ for the longitudinal wave; transducer, 10MHz X-cut quartz
——— $\beta_1 l_1 = 0°$, —·—·— $\beta_1 l_1 = 10°$, ········ $\beta_1 l_1 = 20°$

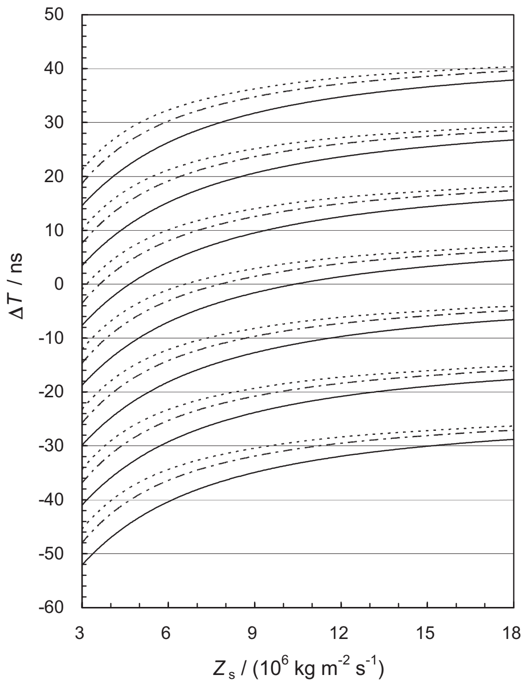

Fig. 3 ΔT as a function of Z_s, n, and $\beta_1 l_1$ for the transverse wave; transducer, 10MHz Y-cut quartz
——— $\beta_1 l_1 = 0°$, —·—·— $\beta_1 l_1 = 10°$, ········ $\beta_1 l_1 = 20°$

Figures 2 and **3** show the relations of ΔT and Z_s for the longitudinal and transverse waves. In order to determine the number of mismatch cycles, Z_s is first calculated with an approximate value for the velocity of sound in which, for example, two echoes are compared with leading edges; this phase comparison usually corresponds to n = +1 or +2. Taking the round-trip travel time to be 10 μs, the mismatch of one cycle (0.1 μs) gives an error of 1% in the velocity of sound and hence in Z_s. If the measured ΔT values fall between ΔT at $\beta_1 l_1 = 0°$ and ΔT at $\beta_1 l_1 = 10°$ or at $\beta_1 l_1 = 20°$ at each n, then the condition $n = 0$ is found. The condition of $n = 0$ must be finally confirmed with the correct Z_s value calculated from the correct value of the velocity of sound. When T is measured at ν_r, then $\gamma/(360\nu)$ in eq. (4) is negligible compared with τ.[1,2)]

4. Velocity of sound in silica glass

Table I shows the number, n, of mismatch cycles determined from the measured values of ΔT, where a silica glass rod is used as the specimen and the physical properties of silica glass are given in **Table II**. From the relations of ΔT and Z_s given by Figs. 2 and 3, it can be found that all the ΔT values fall between $\beta_1 l_1 = 0°$ and $\beta_1 l_1 = 10°$ at each n both for the longitudinal and transverse waves.

Table II shows the velocity of sound in silica glass at 298K. The velocities measured by the present authors are in close agreement, within 0.1%, of those of McSkimin both for the longitudinal and transverse waves.

Table I Number, n, of mismatch cycles

n	$T(\nu_r)$/μs	$T(0.9\nu_r)$/μs	ΔT/ns
Longitudinal wave			
+2	6.8787	6.8885	+9.8
+1	6.7798	6.7785	−1.3
0	6.6812	6.6692	−12.0
−1	6.5815	6.5589	−22.7
Transverse wave			
+2	10.7869	10.7959	+9.0
+1	10.6886	10.6865	−2.1
0	10.5897	10.5764	−13.3
−1	10.4908	10.4669	−23.9

Table II Velocity of sound in silica glass at 298 K

Length of specimen: l_s = 19.925mm
Mass density[6)] = 2.203gcm^{-3}
Velocity of sound reported by McSkimin[6)]
Longitudinal wave: c_l = 5.97kms^{-1}
Transverse wave: c_t = 3.76kms^{-1}
Velocity of sound (present measurement)
Longitudinal wave: $c_l = 2l_s/(T(\nu_r)$ at n=0) = 5964ms^{-1}
Transverse wave: $c_t = 2l_s/(T(\nu_r)$ at n=0) = 3763ms^{-1}
Characteristic impedance (present measurement)
Longitudinal wave: $Z_s = 13.14\times10^6$ kg m^{-2} s^{-1}
Transverse wave: $Z_s = 8.29\times10^6$ kg m^{-2} s^{-1}

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