2E6-1

Proceedings of Symposium on Ultrasonic Electronics, Vol. 33 (2012) pp. 279-280
13-15 November, 2012

Sound Speed Profile Estimation by Single Sonar
単一ソーナーによる音速プロファイル推定

Hisashi Shiba[†] (Radio Application Division, NEC Corporation)
斯波 尚志[†] (NEC 電波応用事業部)

1. Sound Speed Profile Measurements

Sound profiles are very importtant for sonar operations, since the speed distributions affect the sound propagation largely. Usually speed profiles are estimated using tempreture, salinity and density distributions with devices like XBTs and CTDs. Although these are not expensive, most of researchers must follow usage limitations from financial reasons. Moreover, such devices do not measure speed distributions directly.

On the other hand, the acoustical tomography is very popular for investigating ocean structures. This methodology measures sound speed directly by multiple sonar systems. However, it is difficult to deploy and dismantle tomographic systems frequently during usual sonar operations. And the costs sometimes exceed the budget of small sized experiments.

Although a couple of researches utilizing surface measurements for estimting profiles[1-2] are applicable to downsize the operation effort, their accuracies should be verified deeply. For more convinience, a new approach is proposed in this presentation. The new method is achieved by only single sonar. Therefore, it is useful for budegt limited small experiments.

2. A New Estimation Approach

A simple sonic ocean structure model is built for the new sound speed profiles estimation. At first, the ocean is divided into N multiple horizontal layers virtually with ideally flat surface as in Fig.1. The thickness and the sound speed of the Nth layer are d_N and c_N. It is assumed that the speeds do not vary horizontally. Sounds are transmitted by M multiple angles from the sonar on the seafloor. A ray model is applied for the sound propagation in this concept. The incidence angle from the Nth layer to the $(N+1)th$ layer or the outgoing angle from the $(N-1)th$ layer to the Nth layer is $\theta_{M,N}$ for the Mth transmission angle.

The main concept is that the sonar is located on the seafloor and measures echo timings from the surface for various transmission angles. This model is based on that the beam widths are narrow and the sea surface scatters sounds to various directions including incident directions.

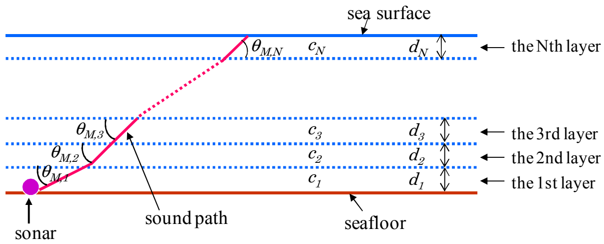

Fig. 1 Layer model configuration.

One way timing T_M from the sonar to the sea surface is

$$T_M = \sum_{i=1}^{N} \frac{d_i}{c_i \sin\theta_{M,i}} \quad (1)$$

for the Mth transmission angle. From Snell's law the angles in the sines are converted by

$$\frac{\cos\theta_{M,1}}{c_1} = \frac{\cos\theta_{M,i}}{c_i} \qquad i = 2,3,\ldots,N. \quad (2)$$

Equation (1) is rewritten as

$$T_M = \sum_{i=1}^{N} \frac{d_i}{c_i\left\{1-\left(\frac{c_i}{c_1}\right)^2 \cos^2\theta_{M,1}\right\}^{1/2}} . \quad (3).$$

It is worthy of note that unknown parameters are only sound speeds in the equation (3).

3. Nonlinear Numerical Solutions

If the echo timings are measured for N angles, N non-linear simultaneous equations are obtained from equation (3), and they are solvable numerically. N equations supply of seeking N unknown sound speeds.

If equation (3) is modified as

$$f_n(\mathbf{c}) = f_n(c_1, c_2, \cdots, c_N) = \sum_{i=1}^{N} \frac{d_i}{c_i\left\{1-\left(\frac{c_i}{c_1}\right)^2 \cos^2\theta_{n,1}\right\}^{1/2}} - T_n , \quad (4)$$

the simultaneous equations becomes like

$$\mathbf{f}(\mathbf{c}) = (f_1(\mathbf{c}), f_2(\mathbf{c}), \cdots, f_N(\mathbf{c}))^{\mathrm{T}} = \mathbf{0} . \quad (5)$$

The right shoulder letter "T" means a transpose. Then the (*k+1*)th solution by Newton-Rapson method becomes

$$\mathbf{c}_{k+1} = \mathbf{c}_k - \left\{ \frac{\partial \mathbf{f}(\mathbf{c}_k)}{\partial \mathbf{c}} \right\}^{-1} \mathbf{f}(\mathbf{c}_k) \ . \tag{6}$$

In this equation, $\mathbf{c}_k$ is defined like

$$\mathbf{c}_k = (c_{1,k}, c_{2,k}, \cdots, c_{N,k})^{\mathrm{T}} , \tag{7}$$

and

$$\frac{\partial \mathbf{f}(\mathbf{c})}{\partial \mathbf{c}} = \begin{pmatrix} \frac{\partial f_1(\mathbf{c})}{\partial c_1} & \frac{\partial f_1(\mathbf{c})}{\partial c_2} & \cdots & \frac{\partial f_1(\mathbf{c})}{\partial c_N} \\ \frac{\partial f_2(\mathbf{c})}{\partial c_1} & \frac{\partial f_2(\mathbf{c})}{\partial c_2} & \cdots & \frac{\partial f_2(\mathbf{c})}{\partial c_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_N(\mathbf{c})}{\partial c_1} & \frac{\partial f_N(\mathbf{c})}{\partial c_2} & \cdots & \frac{\partial f_N(\mathbf{c})}{\partial c_N} \end{pmatrix} . \tag{8}$$

The matrix (8) always has its inverse even if the average sound speeds are equal to each other except right angle incidences. It is not difficult to calculate all these elements sequentially. Practically, the elapse times from the surface to the receiver should be the averages of multiple measurements to suppress the sea surface fluctuation influences.

4. Linear Approximation Solutions

It is not favorable to derive inverse matrices iteratively and to apply unclear convergent conditions to the numerical method like above form the computing viewpoint. Therefore, another method without iteration is proposed based on an approximation referring that the sound speed deviations are small relative to the sound speed itself.

The *m*th term in the summation of the equation (4) is modified to

$$\tau_{n,m} = \frac{d_m}{c_m \left\{ 1 - \left(\frac{c_m}{c_1} \right)^2 \cos^2 \theta_{n,1} \right\}^{1/2}}$$

$$= \frac{d_m}{c_m \left\{ 1 - \left(\frac{c_m}{c_1} \right)^2 + \left(\frac{c_m}{c_1} \right)^2 \sin^2 \theta_{n,1} \right\}^{1/2}} . \tag{9}$$

Supposing the approximation condition

$$\left| 1 - \left(\frac{c_m}{c_1} \right)^2 \right| < \left| \left(\frac{c_m}{c_1} \right)^2 \sin^2 \theta_n \right| , \tag{10}$$

the equation (9) is approximated to

$$\tau_{n,m} \approx \frac{d_m}{c_m \left(\frac{c_m}{c_1} \right) \sin \theta_{n,1}} \left\{ 1 - \frac{1 - \left(\frac{c_m}{c_1} \right)^2}{2 \left(\frac{c_m}{c_1} \right)^2 \sin^2 \theta_{n,1}} \right\} \tag{11}$$

by the 1st order of the equation (10). This approximation produces the following simultaneous equations by 2*N*–1 measurements.

$$\mathbf{\Theta} \cdot \mathbf{\Xi} = \mathbf{T} \tag{12}$$

where

$$\mathbf{\Theta} = \begin{pmatrix} \frac{d_1}{\sin \theta_1} & \frac{d_2}{\sin \theta_1} & \frac{d_2}{\sin^3 \theta_1} & \cdots & \frac{d_N}{\sin \theta_1} & \frac{d_N}{\sin^3 \theta_1} \\ \frac{d_1}{\sin \theta_2} & \frac{d_2}{\sin \theta_2} & \frac{d_2}{\sin^3 \theta_2} & \cdots & \frac{d_N}{\sin \theta_2} & \frac{d_N}{\sin^3 \theta_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{d_1}{\sin \theta_{2N-1}} & \frac{d_2}{\sin \theta_{2N-1}} & \frac{d_2}{\sin^3 \theta_{2N-1}} & \cdots & \frac{d_N}{\sin \theta_{2N-1}} & \frac{d_N}{\sin^3 \theta_{2N-1}} \end{pmatrix}$$

$$\mathbf{\Xi} = \left(\frac{1}{c_1}, \frac{1}{c_2 \left(\frac{c_2}{c_1} \right)}, -\frac{\left\{ 1 - \left(\frac{c_2}{c_1} \right)^2 \right\}}{2 c_2 \left(\frac{c_2}{c_1} \right)^3}, \cdots, \frac{1}{c_N \left(\frac{c_N}{c_1} \right)}, -\frac{\left\{ 1 - \left(\frac{c_N}{c_1} \right)^2 \right\}}{2 c_N \left(\frac{c_N}{c_1} \right)^3} \right)^{\mathrm{T}}$$

$$\mathbf{T} = (T_1, T_2, \cdots, T_{2N-1})^{\mathrm{T}} \quad .$$

Major differences from the numerical solutions in the previous section are that nearly double equations are required and each layer thickness must be different for the inversion of the matrix $\mathbf{\Theta}$. The nearly double equations means nearly double measurements are needed.

5. Conclusions and Perspectives

A new method is proposed for sound speed profile estimation by single sonar. And a new approximation is also presented. Although it seems that this new method is effective for cost cutting, further investigation is needed for practical use. One of the most noticeable points is the accuracy considering beam sizes, surface fluctuations and layer numbers. The accuracy must be evaluated through theoretical analyses or computer simulations.

References

1. J. C. Park and R. M. Kennedy: *IEEE J. Ocean. Eng.,* vol. 21, no. 2, (1996)208.
2. S. Jain and M. M. Ali: *IEEE Geosci. Remote Sens. Lett.*, vol. 3, no. 4, (2006)467.