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Acoustic Simulation Using Wave Equation FDTD (WE-FDTD) Method with Compact FDs

コンパクト差分を用いた WE-FDTD 法による音波伝搬シミュレーション

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1. Introduction

To date, time domain numerical analysis of acoustic fields has become investigated widely as a result of recent computational progress. Some techniques have been proposed as an acoustic field calculation method; the finite difference time domain (FD-TD) method is very widely used for time domain numerical analysis. Many numerical analyses of sound propagation by the FD-TD method have been reported for a few decades.

The standard FD-TD method based on Yee's algorithm, however, causes numerical dispersion error due to using second order finite difference (FD) approximation. To overcome this problem, FD-TD methods using higher order spatial FD and have been proposed. Moreover, compact FD was developed as a more accurate schemes for numerically solving differential equations[1].

The values of the spatial derivatives $f'(i\Delta x)$ on the discretized grids fundamentally determine the accuracy of calculation in the FD-TD simulation. Higher-order FDs yield superior accuracy in exchange for a slightly more complicated formulation. In particular, a tridiagonal (or pentadiagonal) linear system could be solved to calculation cost is a bottleneck in the development of acoustic simulation using the compact FD-TD method.

On the other hand, finite difference time domain method by means of wave equation formulation also has been reported. This is called wave equation finite difference time domain (WE FD-TD) method[2]. The calculation process of this method doesn't use the particle velocity; it uses only the sound pressure. Hence, the required memory can be smaller than conventional FD-TD methods.

In this study, we combine the WE-FDTD method and compact FDs for the second dirivative. The wave equation compact finite difference time domain (WE CFD-TD) method don't also require to calculate the particle velocity; it can recude the calculation time and memory. Furthermore, for acceleralation of simulation, we employ the recursive filtering algorithm[3] and the graphics processing unit (GPU) computing.

2. WE-FDTD method

We present formulation of the WE-FDTD method in the three-dimensional (3D) simulation. The governing equations for linear acoustic fields are given in Eq. (1) and Eq. (2).

$$\rho \frac{\partial}{\partial t}\vec{v} = -\frac{\partial}{\partial x} p \qquad (1)$$

$$\frac{\partial}{\partial t} p = -K \frac{\partial}{\partial x}\vec{v} \qquad (2)$$

In those equations, ρ denotes the density of the medium, K is the bulk modulus, p is the sound pressure, $\vec{v}$ is the particle velocity. Here we assume that the calculation is for a lossless and homogeneous medium.

On the other hand, wave equation of acoustic fields is given as

$$\frac{\partial^2 p}{\partial t^2} = \frac{K}{\rho}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right) \qquad (3)$$

We can obtain Eq. (4) from Eq. (3) by employing second-order central difference approximation in time on collocated grids.

$$p^{n+1}(i,j,k) = 2p^n(i,j,k) - p^{n-1}(i,j,k) + \frac{K\Delta t^2}{\rho}\left\{ \left.\frac{\partial^2 p}{\partial x^2}\right|_i^{n+\frac{1}{2}} + \left.\frac{\partial^2 p}{\partial y^2}\right|_j^{n+\frac{1}{2}} + \left.\frac{\partial^2 p}{\partial z^2}\right|_k^{n+\frac{1}{2}} \right\} \qquad (4)$$

where Δx and Δt respectively denote the grid size and the time step; and $p^n(i)$ represents p (sound pressure) at the time $n\Delta t$ on the grid point $x = i\Delta x$.

3. Acoustic Simulation Using Compact FD-TD Method

We can apply the compact FDs to calculate $\partial p/\partial x$ and $\partial^2 p/\partial x^2$. The compact FDs are obtained by relation equations of the surround values and their derivatives; the relation in the FDTD staggered grid system is given by the following equation:

$$\beta f'_{i-2} + \alpha f'_{i-1} + f'_i + \alpha f'_{i+1} + \beta f'_{i+2} = c\frac{f_{i+5/2} - f_{i-5/2}}{5\Delta} + b\frac{f_{i+3/2} - f_{i-3/2}}{3\Delta} + a\frac{f_{i+1/2} - f_{i-1/2}}{\Delta} \quad (5)$$

where Δ is the grid size of FDTD calculations. (For example, $a = 12/11$, $\alpha = 1/22$ for the fourth-order compact FDs). On the other hand, the relation in the WE-FDTD collocated grid system is given by the following equation:

$$\beta f''_{i-2} + \alpha f''_{i-1} + f''_i + \alpha f''_{i+1} + \beta f''_{i+2} = c\frac{f_{i+3} - f_i + f_{i-3}}{9\Delta^2} + b\frac{f_{i+2} - f_i + f_{i-2}}{4\Delta^2} + a\frac{f_{i+1} - f_i + f_{i-1}}{\Delta^2} \quad (6)$$

where $a = 12/10$, $\alpha = 1/10$ for the fourth-order compact FDs. The parameters of eqs. (5) and (6) determined by relation equations of values and their derivatives using Taylor series. As shown in these equations, the fourth-order compact FDs require the computation of real symmetric tridiagonal matrices.

4. Result and discussion

We show the numerical results obtained using above WE C-FD-TD analysis. Calculation parameters are: the direction of acoustic field propagation, $\pm x$ (1D analysis); grid size, $\Delta x = 0.05$ m; number of grid points, $N_x = 20000$.

Figure 1 shows the sound pressure distribution obtained using C-FDTD and WE C-FD-TD analysis at t = 0.81 s, where $\rho = 1.21$ kg/m^3 and K = 1.4236×10^5 Pa. Here, the initial pressure at t = 0 is given as $p = e^{-\alpha(x-x0)^2}$ [N/m^2]. In this equation, α =1/40 and x_0 = 10000. Two waveforms are very similar. Here, the results of FDTD and WE-FDTD analysis are also plotted in Fig.1. Two waveforms are also similar and numerical dispersion error is caused.

Next, we discuss calculation time of the C-FD-TD method and WE C-FD-TD method. Table 1 shows comparison of the calculation time of fourth-order compact FDs using CPU and GPU.

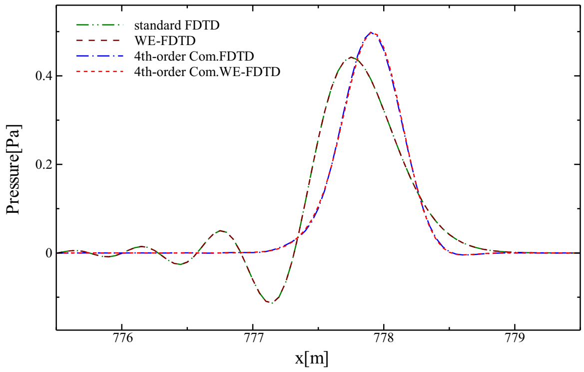

Fig. 1 Distribution of the sound pressure at t = 0.81 s; illustrates standard FDTD (Δt =1.35×10^{-4} , CFL=0.926),WE-FDTD(Δt =1.35×10^{-4} , CFL=0.926), 4th order compact FD-TD (Δt =2.4×10^{-5}, CFL=0.15), 4th order WE-compact FD-TD (Δt =2.4×10^{-5}, CFL=0.15)

Table 1 Calculation Time

Method	Calculation Time [s]
4th C-FD-TD(CPU)	111.52
4th C-FD-TD(GPU)	9.09
4th WE C-FD-TD(CPU)	56.42
4th WE C-FD-TD(GPU)	4.64

Here, in these analyses we estimated the calculation time required for a 3-D simple acoustic model except for the absorbing boundaries. Table 1 shows results of the calculation time between the GPU and CPU results, where the analysis region is 128 × 128 × 128 cells and whole calculation time is divided into 1000 time steps. Here, all measurements are made on NVIDIA Geforce GTX 580 GPU and Intel Core i7 processor 930 2.80GHz (Compiler; Microsoft C/C++ compiler Ver.15.00 for x64). WE C-FD-TD results using CPU is ca. 2 times faster than CPU calculation. Moreover, the results using GPU is ca. 12 times faster than CPU results.

5. Conclusion

This study examines the fast and efficient calculation method of WE FD-TD with compact FDs. Additionally, we made an examination on decreasing the calculation time of these methods using GPU computing.

References

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