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Second Harmonic Component in Focused Gaussian Beam Propagating through a Nonuniform Sound-Speed Layer
音響不均一層を伝搬する集束ガウスビームの第２高調波成分

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1. Introduction

A C-mode imaging system for the nonlinearity parameter *B*/*A* in a thin biological sample has previously prsented with the finite amplitude method using a focused Gaussian beam transmitting through the sample [1]. Since the principle of the measurement premisses uniform samples [2], the nonuniform *B*/*A* contradicts the theory and thereby is not fully reliable. By analyzing the propagation of the focused Gaussian beam within a layer of nonuniform sound-speed, the uncertainty of the *B*/*A* measurement is investigated.

2. Analytical Model

As shown in **Fig.1**, an inhomogeneous layer (II) is set in a focused Gaussian beam formed in water. The K-Z equation is satisfied even in Region II.

$$\nabla^2 p - \frac{1}{c'^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho_0 c'^4}\frac{\partial^2 p^2}{\partial t^2} \quad (1)$$

where $1/c' \approx [1-\delta x/L+\varepsilon(|z|-L)/L]/c$. δ and ε model the sound speed variation in the *x* and *z* directions, respectively. Since the actual measurement employs the sample with a thickness *L* put in front of a reflector, the *c* variance is assumed to be symmetric with respect *z*=0. To omit the reflection at the boundaries, the difference of the characteristic impedance between different regions is set minimal.

In Region I, the successive approximation solution for the fundamental and second harmonic amplitudes P_1 and P_2 has been obtained [3].

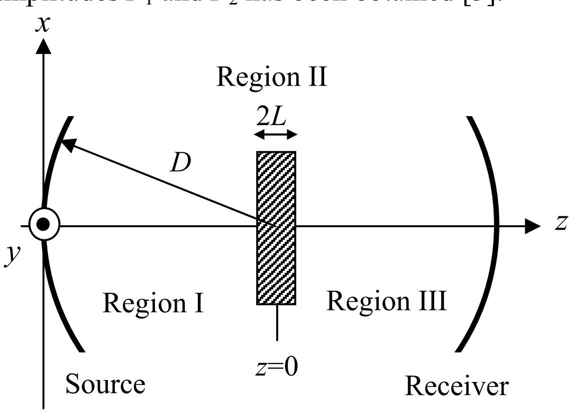

Fig. 1. Model of the measurement system.

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In Region II, the successive approximation of eq.(1) leads to

$$\nabla_\perp^2 P_1 + j2k\frac{\partial P_1}{\partial z} - 2\frac{\delta x}{L}P_1 + 2\varepsilon\frac{|z|-L}{L}k^2 P_1 = 0 \quad (2)$$

$$\nabla_\perp^2 P_2 + j4k\frac{\partial P_2}{\partial z} - 8\frac{\delta x}{L}k^2 P_2 + 8\varepsilon\frac{|z|-L}{L}k^2 P_2 = \frac{2\beta[1-4\delta x/L+4\varepsilon(|z|-L)/L]k^2}{\rho_0 c^2}P_1^2 \quad (3)$$

3. Sound-Speed Variation Normal to Axis

When ε=0, $P_1=P_{10}\exp[-j\varphi_1(z)\delta x]$ is assumed taking into account the phase advance of the sound passing through the higher sound-speed portion. Substituting this expression in eq.(2), the following equations are derived for the terms of δ^0 and δ^1, respectively.

$$\nabla_\perp^2 P_{10} + j2k\frac{\partial P_{10}}{\partial z} = 0 \quad (4)$$

$$-j2\varphi_1\frac{\partial P_{10}}{\partial x} + 2kxP_{10}\frac{\partial \varphi_1}{\partial z} - \frac{2x}{L}k^2 P_{10} = 0 \quad (5)$$

Substituting the solution of eq.(4) for P_{10} in eq.(5), one obtains

$$\frac{\partial \varphi_1}{\partial z} + 2g(z)\varphi_1 - \frac{k}{L} = 0 \ , \quad (6)$$

where $g(z)=\Xi/[2h(z)]$, $\Xi=2\xi+jk/D$, $h(z)=\Xi(z+D)-jk$ and $1/\xi^{1/2}$ is the characteristic radius of the source. The solution for φ_1 yields the following P_1.

$$P_1 = -j\frac{k\gamma(z)P_0}{h(z)}\exp[-jk\frac{z+L}{2L}\frac{h(z)+h(-L)}{h(z)}\delta x] \times \exp[jkg(z)\rho^2] \quad (7)$$

where $\rho^2=x^2+y^2$, $\gamma(z)=\exp[-\xi\delta^2k^2v^2/4D^2|h(z)|^2]$ and $v=(z+L)z(z-L+2D)/L-2(z+D)(z-L)$. Substituting the above P_1 in eq.(3), one obtains

$$\nabla_\perp^2 P_2 + j4k\frac{\partial P_2}{\partial z} - 8\frac{\delta x}{L}k^2 P_2 = -\frac{2\beta P_0^2 k^4}{\rho_0 c^2}\frac{(1-4\delta x/L)}{[h(z)]^2}\exp[-jk\frac{z+L}{L}\frac{h(z)+h(-L)}{h(z)}\delta x] \times \exp[j2kg(z)\rho^2] \quad (8)$$

Using the paraxial approximation of the Green's function [4], eq.(8) is similarly solved as follows.

$$P_2 \approx j\frac{\beta_W k^3[\gamma(z)]^2 P_0^2}{2\rho_0 c^2 \Xi h(z)}[\beta_W \ln\frac{h(-L)}{h(-D)}+\beta\ln\frac{h(z)}{h(-L)}]$$
$$\times\exp[-jk\frac{z+L}{L}\frac{h(z)+h(-L)}{h(z)}\delta x]\exp[j2kg(z)\rho^2] \quad (9)$$

By connecting the solutions for the acoustic pressure continuously at the II-III boundary, the pressure in Region III is finally derived as follows.

$$P_1 = -j\frac{k\Gamma(z)P_0}{h(z)}\exp[-j2k\frac{h(0)}{h(z)}\delta x]\exp[jkg(z)\rho^2] \quad (10)$$

$$P_2 = j\frac{k^3[\Gamma(z)]^2 P_0^2}{2\rho_0 c^2 \Xi h(z)}[\beta_W \ln\frac{h(-L)h(z)}{h(-D)h(L)}+\beta\ln\frac{h(L)}{h(-L)}]$$
$$\times\exp[-j4k\frac{h(0)}{h(z)}\delta x]\exp[j2kg(z)\rho^2] \quad (11)$$

where $\Gamma(z)=\exp[-4\xi\delta^2k^2z^2/|h(z)|^2]$.

Using the geometry and frequency of the actual measurement system, the calculated P_2 distribution for $\delta=0.05$ is compared with that for $\delta=0$ at $z=D$ in **Fig. 2**. Due to the non-uniform sound speed, the beam is refracted. Although the magnitude of P_2 does not change due to the refraction, the receiving signal that is the integration Q_1 and Q_2 of P_1 and P_2 on the receiver surface can vary with δ as shown in **Fig. 3**. It is surmised that the *B*/*A* is underestimated by the half of the rate of the sound speed change in the range equivalent to the sample thickness.

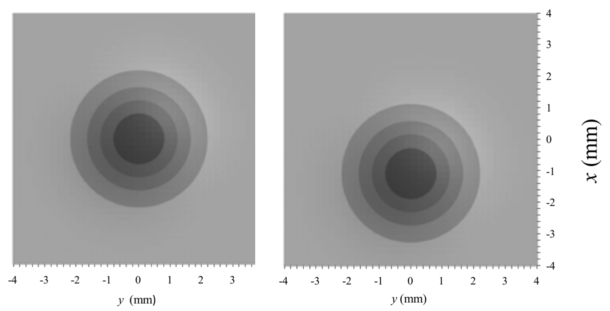

Fig. 2. Influence of sound speed variance normal to the axis on the beam pattern at $z=D$.

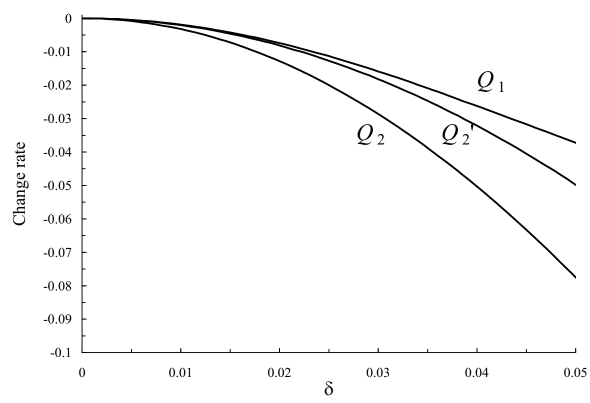

Fig. 3. Change of signal amplitude due to δ. Q_2' is the amplitude of linearly transmitted second harmonic.

4. Sound-Speed Variation Along the Axis

When $\delta=0$, assuming

$$P_n = P_n'\exp(jn\varepsilon k\int_{-L}^{z}\frac{|z'|-L}{L}dz')\ (n=1,2)\ , \quad (12)$$

eqs.(2) and (3) are reduced to

$$\nabla_\perp^2 P_1' + j2k\frac{\partial P_1'}{\partial z}=0 \quad (13)$$

$$\nabla_\perp^2 P_2' + j4k\frac{\partial P_2'}{\partial z}=\frac{2\beta[1+4\varepsilon(|z|-L)/L]k^2}{\rho_0 c^2}P_1'^2 \quad (14)$$

After solving eqs.(13) and (14), P_1 and P_2 continuous at the II-III boundary can be derived as

$$P_1 = -j\frac{kP_0}{h(z)}\exp(-j\varepsilon kL)\exp[jkg(z)\rho^2] \quad (15)$$

$$P_2 = j\frac{k^3P_0^2\exp(-j2\varepsilon kL)}{2\rho_0c^2\Xi h(z)}[\beta_W\ln\frac{h(-L)h(z)}{h(-D)h(L)}+\beta\ln\frac{h(L)}{h(-L)}]$$
$$\times\exp[j2kg(z)\rho^2]$$
$$+j\frac{2\varepsilon\beta k^3P_0^2\Lambda\exp(-j2\varepsilon kL)}{\rho_0c^2Lh(z)}\exp[j2kg(z)\rho^2] \quad (16)$$

where $\Lambda=\frac{h(-L)}{\Xi^2}\ln\frac{h(0)}{h(-L)}-\frac{h(L)}{\Xi^2}\ln\frac{h(L)}{h(0)}$.

As shown by the solid line in **Fig. 4**, P_2 decreases with the increase of the sound speed due to ε. Then the *B*/*A* is underestimated by the same rate as ε.

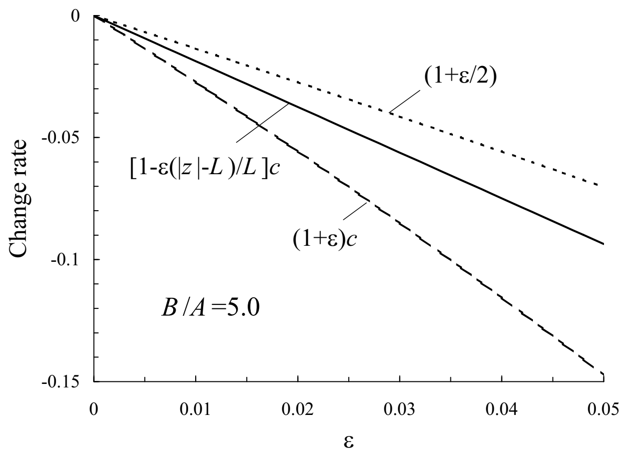

Fig. 4. Change of second harmonic amplitude; — nonuniform sound-speed in the axial direction, … , ---uniform sound-speed

5. Conclusion

When the variance of $1/c' \approx [1-\delta x/L+\varepsilon(|z|-L)/L]/c$ is assumed, the underestimation of *B*/*A* by the rate of $\varepsilon+\delta/2$ is predicted. This error is relatively small.

References

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