Estimation of Gas Void Fraction in Gas-Bubble-Contained Sand with Difference Frequency Acoustic Wave

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1. Introduction

When two primary acoustic waves of different frequencies are incident on gassy sediments, nonlinear acoustic waves can be strongly generated at the difference frequency of the primary frequencies¹). The nonlinearity parameter of gassy sediment is related to the gas void fraction provided the primary frequencies are lower than bubble resonance frequencies¹).

In this study, the nonlinearity parameter of gas-bubble-contained sand was evaluated by using a theory of the parametric acoustic array for difference frequency acoustic wave. Gas void fraction of the gas-bubble-contained sand was estimated from the evaluated nonlinearity parameter.

2. Theory

If we consider that two primary acoustic waves are incident on a gas-bubble-contained sand layer with thickness l in water as shown in **Fig.1**, the generation of the nonlinear acoustic wave at the difference frequency can be described by using nonlinear wave equation²). When the acoustic pressure fields of two primary acoustic waves in the gas-bubble-contained sand layer located at far distance H are expressed by

$$P_{1,2} = \frac{A_{1,2}TR_{1,2}/D_{1,2}(\theta')}{H} \exp\{-\alpha_{1,2s}(z-H)\} , \quad (1)$$
$$\times \exp[j\{\omega_{1,2}t - k_{1,2w}(z + \frac{\rho_{\perp}^{2}}{2H})\}]$$

the nonlinearity parameter of the layer from the solution of nonlinear wave equation²) for the difference frequency acoustic wave can be given as follows:

$$\varepsilon = \frac{-j4p_d \rho_s c_s^{3} H^2 \exp(-\alpha_{ds} l)(2jk_{dw} + \alpha_{Ts})}{\omega_d T' T^2 A_1 A_2 R_{f1} R_{f2} [1 - \exp\{-(2jk_{dw} + \alpha_{Ts})l\}] \{1 - \exp(-j\frac{k_{dw} \rho_{\perp}^{*2}}{H})\}}$$
(2)

where $A_{1,2}$ are pressure amplitudes of two primary waves at the piston source, c_s and ρ_s are the sound speed and the density of the sediment layer $D_{1,2}(\theta')$ and $R_{1,2f}$ are the beam directivity functions and the

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Fraunghoffer lengths for primary frequencies at the

piston source in water, $k_{1,2w}$ and k_{dw} are the wave numbers of primary and difference frequency acoustic waves in water, respectively, *T* and *T'* are the transmission coefficients of the acoustic waves between water and the gas-bubble-contained sand layer. ρ_{\perp}^* is a radius of the interaction zone of primary waves in the layer, $\alpha_{Ts} = \alpha_1 + \alpha_2 - \alpha_{ds} / \cos \phi$ is the combined attenuation coefficient of primary and difference frequency acoustic waves.



Fig. 1 Schematic diagram for theoretical description.

If the primary frequencies are lower than resonance frequencies of the bubbles in the pore water of the layer, the gas void fraction β in the layer can be simply expressed with the nonlinearity parameter as in water containing gas bubbles as follows¹:

$$\beta = \frac{2\varepsilon\gamma^2 P_0^2}{\rho_w^2 c_w^4 (\gamma + 1)}$$
(3)

where $\gamma = 1.4$ is the polytropic exponent of the bubble gas, $P_0=10^5$ Pa.

2. Experimental Setup

A schematic diagram of the experimental setup for acoustic measurements is shown in **Fig. 2**. The gas-bubble-contained sand layer was located at far distance of 900 mm from the transducer (RESON TC2122). The diameter of transducer was 180 mm and its center frequency was 33 kHz. The transducer was simultaneously driven at two primary frequencies of 28 and 33 kHz to get the difference frequency of 5 kHz. The acoustic pressure amplitude for the primary acoustic wave of 28 kHz at the transducer was 58 kPa. It was 53 kPa for primary acoustic wave of 33 kHz. The reflected signals from the layer were received by a hydrophone (B&K 8103). The hydrophone was placed at a distance of 900 mm from the layer. To measure the sound speed and the acoustic attenuation in the sediment layer, another hydrophone (EDO 6600) was installed at the distance of 25 mm in forward direction of the sediment layer. The received signals were acquired using a 300-MHz digital storage oscilloscope (LeCroy LT342) and stored on a computer for off-line analysis.



Fig. 2 Schematic diagram of experimental setup

3. Results and Discussion

Fig. 3(a) shows the frequency spectrum of the signal received at a distance of 900 mm from the transmitter without gas-bubble-contained sand in water. As shown in Fig. 3(a), the difference frequency component almost appeared below the background noise level. Fig. 3(b) shows the frequency spectrum of the signal reflected from gas-bubble-contained sand in water. The pressure levels at the primary frequencies of 28 and 33 kHz in Fig. 3(b) were largely decreased in comparison with those in Fig. 3(a). If the primary frequencies were in the bubble resonance frequency ranges, the acoustic energies of the primary waves could strongly be absorbed by the gas bubbles. In this case, the pressure levels at the second harmonic frequencies could also highly be increased compared to other nonlinear frequency components. Therefore, the significant decreases of the primary pressure levels at 28 and 33 kHz and increases of the second harmonic pressure levels at 56 and 66 kHz in Fig. 3(b) could be caused by the resonant gas bubbles at the primary frequencies in gas-bubble-contained sand. These gas bubbles could be produced by a combination of main gas bubbles in making process of the gas-bubblecontained-sand.

The pressure level at the difference frequency of 5 kHz in Fig. 3(b) was 167.9 dB, which was 38 dB higher than the background noise level. This could be caused by the high nonlinearity of the gas-bubble-contained-sand. Its nonlinearity parameter could be determined by using Eq. (2). The nonlinearity parameter of the gas-bubble-contained-sand was estimated as $\varepsilon = 2,539$. In this estimation, the attenuation coefficient of the difference frequency wave was ignored, because it was very small compared to those of the primary waves. Since the resonance frequencies of the main gas bubbles were supposed to be much higher than the primary frequencies, the gas void fraction in gassy sand culd simply be estimated by using Eq. (3) and the estimated nonlinearity parameter. Then, the gas void fraction was estimated as $\beta = 7.98 \times 10^{-4}$. This is consistent with that estimated from the sound speed variation method in gas-bubble-contained- sand³⁾.



Fig. 3 Frequency spectra of (a) the signal received at a distance of 900 mm from the transmitter without the gas-bubble-contained-sand layer and (b) the reflected signal from the layer in water

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