Effects of Electrodes on Frequency-Temperature Characteristics of AT-cut Quartz Crystal Unit

水晶 AT 板振動子の周波数一温度特性におよぼす電極の影響

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1. Introduction

The use of metal electrodes deposited on major surfaces of an AT cut quartz crystal unit for piezoelectric excitation of a thickness-shear mode has been common in many applications. It provides a good electromechanical coupling and eliminates frequency changes due to a relative shift of electrodes with respect to the crystal plate. On the other hand, however, such important charcteristics of a crystal unit as resonant and antiresonant frequencies and Q, and their dependences on temperature and time, are no longer solely determined by porperties of quartz crystal.

Conventional analyses assumed that the thickness of electrodes was thin enough, so that only the mass loading of electrodes were taken into account. The thickness of a crysatal plate, which is inversely proportional to the foundamental resonant frequency and its overtones, has been reduced to meet recent demands for devices operable in a higher frequency range. The thickness of electrodes, however, cannot be proportionally reduced, because adequate conductivity has to be maintained. Then electrodes behave not as simple masses but as continuous bodies, in which elastic waves propagate. This causes much differences between characteristics of the fundamental mode and that of overtones, because the effective thickness of an electrode in terms of wave length proportionally increases with the overtone number.

This paper presents an analysis based on Mason's one-dimensional equivalent circuit of a resonator vibrating in thickness-shear mode. Both plateback, which is a decrease in a resoanant frequency, and changes in the first order frequency-temperature coefficient are derived as functions of the thickness of electrodes and the overtone number. Conventional analyses taking only the mass-loading into account yielded little variations in both characters.

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AT-cut resonators with various thickness of electrodes were fabriacatied and measured up to the 7th overtone. Experimental results were in a good agreement with the present analysis.

2. Analysis

Since the lateral dimensions of electrodes are two orders larger than the thickness of a crystal plate, one-dimensional analysis based on Mason's equivalent circuit of a resonator vibrating in thickness-shear mode can be used for a good approximation.

We assume a symmetric arrangement of electrodes with respect to the center plane of the crystal plate. Then the equivalent circuit is reduced to the simplified circuit shown in **Fig. 1**. Mechanical characteristics of an electrode are represented by a transmission line short-circuited at the far end, which is mechanically traction-free.

The input impedance at the electrical terminal is given by (1).



Fig.1 Equivalent circuit of AT-cut with symmetric arrangement of electrodes on major surfaces.

$$Z_{in} = \frac{1}{j\omega C_{D}} \frac{1 - k^{2} \frac{\tan X}{X} - ML \times \tan X \frac{\tan X_{e}}{X_{e}}}{1 - ML \times \tan X \frac{\tan X_{e}}{X_{e}}}$$
(1)

where

$$Z_{1} = j Z_{0} \tan X, \qquad Z_{2} = \frac{Z_{0}}{j \sin 2X}, \qquad Z_{A} = j Z_{e} \tan X_{e}$$
$$Z_{0} = \rho_{q} v_{q}, \qquad Z_{e} = \rho_{e} v_{e}, \qquad X = \frac{\omega h}{v_{q}}, \qquad X_{e} = \frac{\omega l_{e}}{v_{e}}$$

$$ML = \frac{2\rho_e l_e}{\rho_a 2h} : \text{mass loading}$$

 $k^{2} = \frac{\pi \phi^{2}}{\omega_{a0} C_{D} Z_{0}} : \text{ electromechanical coupling factor}$

$$\omega$$
 : angular frequency, $\omega_{a0} = \frac{1 + v_q}{2h}$

h : half thickness of crystal plate

 l_e : thickness of electrode on one surface

 ρ : density, v: sound velocity, $C_{\rm D}$: clumped capacitance (Subscripts q and e stand for quartz and electrode, respectively. All notations refer to an unit area at the surface.)

$$ML = \frac{\frac{\tan X_e}{X_e}}{\frac{1}{X_e}} = \frac{\frac{X}{\tan X} - k^2}{X^2}$$
(2)
$$ML = \frac{\frac{X}{\tan X} - k^2}{X^2}$$
(3) (for thin electrodes)
$$\frac{1}{k^2} = \frac{\tan X}{X}$$
(4) (for very thin electrodes)

The resonance condition is given by (2), in which the right and the left side solely depend on material constants of quartz and electrode, respectively. Reliable values including its temperature coefficients of the former are available. In the latter, however, little data are available except ML, mass loading which is the ratio of mass of electrodes and mass of quartz plate.

For thin electrodes, (2) can be approximated by (3), which is further reduced to (4) for very thin electrodes of negligible mass.

One of the authors (M.O.) tabulated solutions of (4) in 1963 and explained differences in temperature characteristics between resonance and antiresonance in 1969. Bechmann discussed early influence of the order of overtone on temperature characteristics in 1955.

Ballato & Lukaszek conducted an extended numeric analysis based on (3) in 1975. They treated ML as a variable dependent on temperature, because "mass per unit area of electrode coating is constant with respect of temperature. Because ρ and h are functions of temperature."

The product of ρ and h, however, is constant for vertical deformation due to temperature. Electrodes, which are soft and adherent to crystal surface, laterally deform as the crystal plate deforms. Hence mass per unit area of electrode is no longer constant, but ML is still constant.

The present analysis is conducted based on full equation (2), in which ML is considered as independent on temperature. Characteristics of electrodes are represented by X $_{\rm e}$, instead of individual material constants, which are hardly available. The first order temperature coefficient of frequency can be derived by differentiation of (2) with respect to temperature.

$$T_{\mathbf{x}} = \frac{D_{\mathbf{e}}}{D_{\mathbf{x}}} T_{\mathbf{e}} - \frac{D_{\mathbf{k}}}{D_{\mathbf{x}}} T_{\mathbf{k}}$$
(5)

$$T_{x} = \frac{1}{X} \frac{\partial X}{\partial T}, T_{e} = \frac{1}{X_{e}} \frac{\partial X_{e}}{\partial T_{e}}, T_{k} = \frac{1}{k} \frac{\partial k}{\partial T}$$
$$D_{x} = \frac{-\cos X \sin X - X}{X \sin^{2} X} + \frac{2k^{2}}{X^{2}}$$
$$D_{e} = ML * \frac{X_{e} - \cos X_{e} \sin X_{e}}{X_{e} \cos^{2} X_{e}}, D_{k} = -\frac{2k^{2}}{X^{2}}$$

The first term in (5) is dominant in overtones and the second term is dominant only in the fundamental resonance. This feature cannot be derived from (3).

3. Comparison with experiments

AT-cut units at the fundamental frequency of 27MHz with golden electrodes were made and measured up to 7th overtone. Since units were designed for overtone operation, severe activity dips were observed in the fundamental mode, which spoiled the accuracy in that mode.

Fig. 2 shows a comparison of calculated and measured degrease of frequency (plate back) due to mass loading. For thin electrodes it is almost constant regardless of order of overtone, whereas it varies as electrodes become thick.

Fig. 3 shows a comparison of calculated and measured change in the first order temperature coefficient as function of electrode thickness. The change becomes larger in higher overtone.



Fig.2 Calculated (line) and measured (mark) plate back due to mass loading at 25 $^{\circ}$ C.



Fig.3 Calculated (line) and measured (mark) Tx.