# Off－axis Aplanatic Straubel Mirror for Wide Field of View軸外し配置によるアプラナートシュトラウベル反射鏡の広角化 

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## 1．Introduction

Acoustic lenses and mirrors are useful devices for underwater imaging because these devices do not require complex electrical circits． Authors have attempted to design the lenses and mirrors having appropriate shape for imaging．We proposed an aplanatic Straubel（AS）mirror，which was a back surface mirror and could correct spherical and coma aberrations ${ }^{1)}$ ．The AS mirror made of silicone rubber showed less aberration and higher gain than lenses made of silicone rubber in simulation．

The cross－sectional shape of AS mirror and raytrace diagrams are shown in Fig．1（a）．The focal length，$z_{\mathrm{f}}$ ，and diameter are 400 mm and 200 mm ， respectively．The sound speed in water and silicone rubber are regarded as $1500 \mathrm{~m} / \mathrm{s}$ and $1000 \mathrm{~m} / \mathrm{s}$ ， respectively．The AS mirror has a receiver array in front of the mirror as shown in Fig．1（a）．The diameter of receiver array，$D_{\mathrm{R}}$ ，is determined by focal length，$z_{\mathrm{f}}$ ，and angle of view，$\varphi$ ，as following equation．

$$
\begin{equation*}
D_{\mathrm{R}}=2 z_{\mathrm{f}} \tan \frac{\varphi}{2} . \tag{1}
\end{equation*}
$$

Large receiver is effective to obtain wide angle of view because aberrations are generated by the short focal length．However，when the mirror required to be wide angle of view，the receiver becomes large and interrupts the incident sound waves．

An off－axis AS mirror，shown in Fig．1（b），is available to avoid interruption of receiver when the incident angle，$\theta_{x}$ ，is larger than $0^{\circ}$ ．Here，the incident angle，$\theta_{x}$ ，should be discriminated from the angle of view，$\varphi$ ，because angle of view，$\varphi$ ，means actually visible area．The off－axis AS mirror corresponds to the half side of normal AS mirror shown in Fig．1（a）．

Being asymmetry，off－axis mirrors usually have more serious aberrations or problems than normal mirrors ${ }^{2}$ ．The off－axis AS mirror will also has several problems．Thus，it is necessary to survey available angle of view．In this report，we suggest a possibility of wide angle of view using an off－axis


Fig． 1 Cross－sectional shapes and ray trace diagrams：（a）Normal AS mirror and（b）off－axis AS mirror．Gray areas show RTV silicone rubber．Black areas show rigid body．

AS mirror．Converged sound pressure fields of normal and off－axis AS mirrors are calculated and compared．

## 2．Calculation Condition

Optical lenses and mirrors are designed with the ray theory．However，applying the ray theory to evaluate acoustic lenses and mirrors is difficult because ratio of diameter to wavelength is significantly smaller than that of optical devices． Thus，the converged sound pressure fields should be calculated with the wave theory．The Rayleigh integral expression is used for calculation as same as former study ${ }^{2-3)}$ ．

Table I Parameters used for calculation.

| Parameters | Water | Silicone |
| :--- | ---: | ---: |
| rubber |  |  |
| Sound speed $(\mathrm{m} / \mathrm{s})$ | 1500 | 1000 |
| Density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | 1000 | 1490 |
| Attenuation coeficient $(\mathrm{Np} / \mathrm{m})$ | 0 | 57.6 |

The parameters used for calculation are shown in Table. I. The mirror, shown in Fig. 1 in black area, is regarded as a rigid body. When the sound wave passes from water to rubber or from rubber to water, the transmission coefficient is multiplied. Each surface of AS mirrors is divided into about 30,000 areas. The source is regarded as point source and located on $\left(100 \sin \theta_{x}, 0,100\right)(\mathrm{m})$. Thus, the incident wave is almost plane wave. The frequency of sound wave is 500 kHz .

Acoustic shadow is made by the receiver on the mirror. The area of shadow is calculated geometrically. We ignored the thickness of receiver, the multiple reflections between the receiver and mirror, and the diffraction generated by the receiver to simplify the calculation.

## 3. Results

### 3.1. Normal AS mirror

Beam patterns of the normal AS mirror, normalized by the maximum power of $\theta_{x}=0^{\circ}$, are shown in Fig. 2. Calculated area is shown in Fig. 1(a) with a chain line. When the receiver array does not exist, the beam widths and focal powers are almost the same in each incidence angle as shown in Fig. 2(a). However, when the array for $\varphi=10^{\circ}$ exists, the beam becomes wide at $\theta_{x}=10^{\circ}$ and side lobe level increases at $\theta_{x}=0^{\circ}$ as shown in Fig. 2(b). Additionally, focal powers decrease at each incident angle. Although it is possible to correct the intensity or distortion of image at post-process, defocusing and side lobes cannot correct. When the array for $\varphi$ $=15^{\circ}$ exists, the mirror is absolutely covered. Hence, the limit angle is $15^{\circ}$, and problems occur when the angle is $10^{\circ}$.

### 3.2. Off-axis AS mirror

The off-axis AS mirror is designed with the same focal length and diameter as the normal AS mirror as shown in Fig. 1. Beam patterns of the off-axis AS mirror are shown in Fig. 3. Scales of the figure is same as Fig. 2. The incident angle is changed from $0^{\circ}$ to $35^{\circ}$ at intervals of $5^{\circ}$. Side lobes are not generated and the beam widths are almost the same in all incident angles. However, the focal power at $\theta_{x}=0^{\circ}$ is smaller than that of Fig. 2(a), which shows the beam patterns calculated without the receiver array. Additionally, the focal power


Fig. 2 Beam patterns of normal AS mirror: (a) Without receiver array and (b) with receiver array for $\varphi=10^{\circ}$. The receiver does not exist on gray area.


Fig. 3 Beam patterns of off-axis AS mirror. The receiver does not exist on gray area.
decreases when the incident angle becomes large.
In result of calculations, when the image is corrected at post-process, the field of angle of off-axis AS mirror is about $35^{\circ}$. It seems that the off-axis AS mirror has sufficient angle of view because that of optical standard lens is from $25^{\circ}$ to $50^{\circ}$.

## 4. Conclusions

We researched available angle of view of off-axis AS mirror with calculations. The limit angle of normal AS mirror was about $15^{\circ}$, and problems occurred when the angle was $10^{\circ}$ because the receiver array interrupted the incident sound wave. On the other hand, there was a possibility that the limit angle of view of off-axis AS mirror reached $35^{\circ}$ when received image was corrected at post-process.

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