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Extension of Measurement Viscosity Range of EMS System
EMS システムにおける測定粘度域の拡張

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1. Introduction

Visco-elasticity is one of the important properties of soft condensed materials describing its mechanical response. The visco-elasticity is a variable depending on the speed of the medium deformation, of which behavior is described usually in terms of two aspects; the rheological curve and the ultrasonic spectroscopy. These two studies are composed of different measurement techniques, but share the same concept that the viscoelasticity should be represented as the function of the frequency or the inverse of the deformation time.

The rheology has a technical advantage over the ultrasonic spectroscopy that it can measure the small viscosity as low as 10^{-3} Pa s, which is approximately that of pure water. The ultrasonic absorption due to the shear viscosity of water is in the order of 10^{-2} m^{-1} at 1 MHz, that is too small to be accurately observed.

It is also important that the rheology measurement determines the low frequency limit of the ultrasonic propagation in materials with different measurement technique; complementary use of those two techniques is quite effective for the mechanical science of materials. To be honest from the standpoint of the rheologist, however, the measurement of the low viscosity is still difficult also for the rheology at present.

2. Measurement of low viscosity

EMS (Electro-Magnetically Spinning Sphere) viscometer is the recent invention, which supplies us easy measurement of the viscosity depending on the shear deformation rate. We are trying to expand the measurement range towards both lower and higher viscosities. In this paper, we described the application of EMS to the measurement of low viscosities less than 1 mPa·s.

Aqueous solutions of materials are of course the important stuff for the chemical industry as well as the bio-engineering. The viscosity of the solution is generally more or less larger than that of the solvent and the resolution of 10 % at the viscosity η=1 mPa·s would be enough for the application to the above promising industrial fields.

The lower limit of the viscosity measurable by EMS system is determined by the condition that the torque due to the surrounding viscosity overcomes that of the friction between the bottom of the probe sphere and the sample cell. The former is proportional to the third power of sphere radius, while the latter to the fourth, and the condition is satisfied for an enough small sphere. As for pure water at the sphere rotation of 10 rps, radius of 1 mm for aluminum sphere is the boundary of the condition. It shows, in other words, that the viscosity of water cannot be determined with satisfactory accuracy in the present EMS system.

To settle the problem, let us consider the flow field of the medium around the spinning sphere at the bottom of the cell. The steady flow field around the sphere set in infinite medium is analytically given as

$$v_\phi = \Omega \sin\theta (R^3 / r^2)\,,\;\; v_\theta = 0\,,\;\; v_r = 0\,,$$

where R is the sphere radius, Ω the angular velocity of the sphere. The polar coordinate is taken so that θ=0 and π correspond to the sphere bottom and top, respectively. The geometry is schematically shown in Fig.1. The viscous torque applied to the surface area of the sphere between $\theta \sim \theta + d\theta$ is $T(\theta)d\theta = 6\pi\eta R^3 \sin^3\theta d\theta$ and the whole torque is then calculated to $M = 8\pi\eta R^3 \Omega$.

Here, let us consider the effect of the bottom touching with the sphere. In the vicinity of the bottom and θ=0, the sphere surface is approximated to the parabola. The shear rate at the gap of the sphere and the bottom is $r = 2\Omega / \sin\theta$ and the viscous torque is calculated to

$$T'(\theta) = 4\pi\eta R^3 \sin\theta d\theta \quad (\theta \sim 0).$$

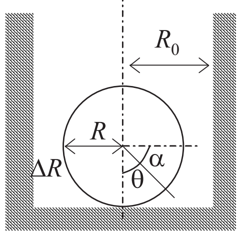

Fig.1 Schematic view of the geometry used in the calculation.

The torque $T'(\theta)$ should be analytically connected to $T(\theta)$ as θ increases towards $\theta \sim \pi/2$.

We can show with the above formulas that the torque applied to the surface area in the vicinity of the sphere equator is dominant as the resistant torque due to the viscosity. The excess amount of $T'(\theta)$ over $T(\theta)$ would give rough estimation of the contribution from the bottom effect to the whole viscosity torque, which is found as large as 1/10 from the numerical calculation. Other important information obtained from the numerical approach to the flow field is that the torque applied in the region of $\pi/4 < \theta < 3\pi/4$ has the portion more than 3/4 of the total torque.

It would, therefore, be a simple idea to emphasize the viscous resistant torque for the low viscosity sample that the shear deformation rate is increased by confining the sphere in the thin cylinder.

The shear deformation rate around the sphere given by $r = (\partial v_\phi / \partial r - v_\phi / r)$ is $r = 3\Omega \sin R^3/r^3$, which reaches as far as the distance R from the sphere surface. Restricting the space of medium flow by inducing thin cylinder with in the gap ΔR from the surface, the apparent torque due to the viscosity would increase by the factor of $(R/\Delta R)$.

Let us roughly estimate the torque applied to the sphere confined in a cylinder with the inner diameter $R + \Delta R$. Here, we ignore the effect of the bottom of the sample cell. From the symmetry, the shear rate at the sphere equator is $\gamma = 2R_0^2/(R_0^2 - R^2)\Omega$, where the angle α is taken as $\alpha = \theta - \pi/2$ and $R_0 = R + \Delta R$.

It would be natural to consider that the shear deformation in the gap between the sphere and the cylinder is dominant to determine the torque. The gap decreases with increasing α and we cut off the summation of the torque at the angle α_c, which gives the gap of $(1+\beta)\Delta R$, β being a constant in the order of unity. The cut off angle α_c is then given by $R(1 - \cos\alpha_c) = \Delta R$. It means that the sphere is approximated to a disk with the radius R and the thickness $D = 2R\sin\alpha_c$, and with the condition of $\Delta R \sim R$, D is given by $D = \sqrt{8R \cdot \Delta R}$. By substituting this relation, we obtain the torque as $T \approx 4\sqrt{2}\pi R^3 \sqrt{(R/\Delta R)}\eta\Omega$

The result shows that the torque increases with decreasing gap as $T \sim (R/\Delta R)^{-1/2}$. Roughly, the value of $\Delta R << R/10$ enhances the viscous torque apparently by the factor of $\sqrt{10}$.

3. Experiment

Here, we briefly describe the principle of the EMS viscometer. A metal sphere is immersed in the sample, which works as a probe of the viscosity. Rotating magnetic field with the magnitude of 100 mT is applied to the sphere and the Lorentz interaction between the induced current and the magnetic field works so that the sphere rotates following the rotation of the magnetic field. The relation between the applied torque and the revolutional speed gives the viscosity as a function of the sphere rate.

To examine the suppression of the rotation of the confined sphere in a small space, we prepared a cylinder with the inner diameter of 2.1 mm made of metal, whereas the sphere is made of aluminum and

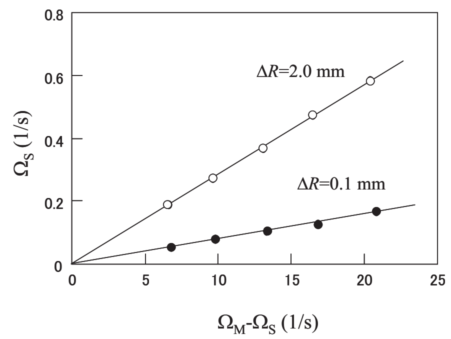

Fig.2 Relation between the sphere rotation and applied torque for the gap of ΔR=0.1 (closed circles) and 2.0 mm (open circles).

has the diameter of 2.0 mm.

Figure 2 shows the rotational speed of the sphere depending on the difference between the revolution of the magnetic field and the sphere, which is proportional to the applied torque. The results obtained for the sphere in a 2.1 mm diameter cylinder and the usual sample tube with the diameter of 6.0 mm are represented by the closed and open circles, respectively. The sample is the viscosity standard liquid of η=1.0 Pa s.

The rotation in the thin cylinder is apparently slower than that in a free space due to the effect of the wall in the vicinity of the sphere surface and the ratio of suppression is bout 3.7, which roughly agrees with the value of $\sqrt{2R/\Delta R} \sim 3$. The result shows that the thin cylinder successfully enhances the torque due to the sample viscosity. It would be then possible to improve the accuracy and the resolution of the measurement for the lowly viscous liquids.

References

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