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Analysis of Frequency Gap of Antenna Structure Resonator Using Mathieu Equation

Mathieu 方程式を用いたアンテナ構造振動子の周波数ギャップの解析

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1. Introduction

The vibration behavior of the antenna structure resonator has been characterized by the relationship between the resonant frequency and its vibrational mode. That is, the vibrational mode changes from the first to the tenth modes, and then repeats again from the first to the tenth modes.[1] Furthermore, the frequency gap, which the resonant vibration does not appear, exists between the first tenth and the second first modes.[1]

In this study, we show that the perturbed time dependent axial force exerts on the elements of the antenna structure resonator by relating the stable or unstable vibrations of the Mathieu equation which shows the parametric vibration and the calculated frequency gap by the Bernoulli-Euler equation [1] applied to the antenna structure resonator.

2. Analysis of the antenna structure resonator using Mathieu equation

We considered the analytical model of the antenna structure resonator which a T-shaped unit was connected in sequential up to twenty one units. As a T-shaped unit combined three flexural beams at one point, the Bernoulli-Euler equation with the time dependent axial force for a flexural beam in the [j] unit is given by [2]

$$EI\frac{\partial^4 w_j(x,t)}{\partial x^4} + P(t)\frac{\partial^2 w_j(x,t)}{\partial x^2} + \rho A\frac{\partial^2 w_j(x,t)}{\partial t^2} = q_j(x,t) \quad (1)$$

where, $w(x,t)$ is the deflection displacement, E is Young's modulus, I is the moment of inertia, $P(t)$ is axial force, ρ is the mass density, A is the cross section, $q(x,t)$ is the external force of the beam, x is the coordinate, t is the time, respectively

Figure 1 shows that one boom and two elements are connected at joint [j] of unit [j] and unit [j-1], x-coordinate, and their sizes. The analysis was carried out using the boundary conditions obtained from the linking conditions of unit [j] and unit [j-1]. The Lorentz force,[3] is the driving force of the antenna structure resonator, drives the elements so

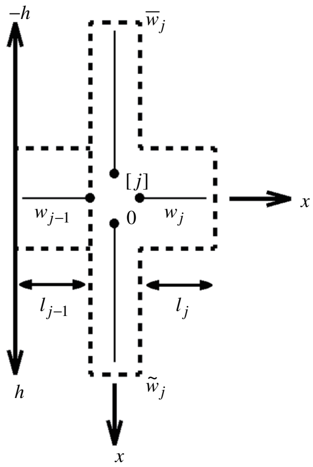

Fig. 1 Connection of unit [j] and unit [j-1] at joint [j]

that the time dependent outward axial force like a centrifugal force yields in the elements. Parameters greatly influenced for the formation of the Mathieu equation derived from applying the Bernoulli-Euler equation with the time dependent axial force to the antenna structure resonator are $\tilde{\Omega}_{nj}^2$, being in proportion to the angular frequency of the system, and $\tilde{\lambda}_{nj}$, being related to the axial force. They are given by

$$\tilde{\Omega}_{nj}^2 = \frac{\rho A h_j^4}{E\tilde{I}}\omega^2 , \; \tilde{\lambda}_{nj} = \frac{P(t) h_j^2}{E\tilde{I}} . \quad (2)$$

where h is the element's length.

As the perturbed term by the axial force is very small, we assume that $\tilde{\lambda}_{nj}$ is very small compared to $\tilde{\Omega}_{nj}^2$. Since the antenna structure resonator can be modeled as a group of a repeated T-shaped structure, the equation of motion describing the vibrational behavior of the antenna structure resonator becomes a similar type of the Mathieu equation depicted below,

$$\ddot{Z}_n(t) + \omega^2[1+\alpha\cdot\cos(\nu t)]Z_n(t) = 0 \quad . \quad (3)$$

where, α is the parameter which changes by both the angular frequency and the axial force.

We obtained the frequency gap by requiring the stable and unstable regions of the vibration of the Mathieu equation derived from the antenna structure resonator. The analysis adopted in this study is the general method [4] used in the analysis of the Mathieu equation.

3. Investigation of frequency gap by calculating the eigenvalue of Mathieu equation

Figure 2 shows the calculated stable and unstable regions of the vibration of the Mathieu equation derived from the antenna structure resonator, with the finite length, having 20 elements on each side of the boom. In Fig. 2, the calculated resonant frequency ω for the eigenvalue δ is obtained in such way that the resonant frequency 20.44 MHz for the first mode of the system is adopted as a driving frequency ν and the axial force is set to be 5.58×10^{-5} N. From Fig. 2, the frequency gap is calculated to be 840 MHz～1.197 GHz.

The frequency gap of the resonant frequency in the antenna structure resonator, with the finite length, having the 20 elements on each side of the boom was calculated to be 763 MHz～1.197 GHz using Bernoulli-Euler equation. [1] On the other hand, the frequency gap was calculated to be 927 MHz～1.197 GHz using the transfer matrix method for the flexural wave propagating through the infinite antenna structure resonator under the periodic boundary condition. [5] It was found that the calculated frequency gap using the Mathieu equation was almost equal to the calculated frequency gap using the Bernoulli-Euler equation.

Figure 3 shows the relationship between the stable and unstable regions of the vibration of the Mathieu equation when we set the magnitude of the axial force to be 5.58×10^{-10} N. As shown in Fig. 3, δ becomes constant regardless of ω and there is no unstable region. Therefore, as for the magnitude of the axial force being 5.58×10^{-10} N, the Bernoulli-Euler equation with the time dependent axial force is almost equivalent to the Bernoulli-Euler one without that. From these calculations, the antenna structure resonator was found to show the parametric vibration induced by the time dependent axial force exerted on the elements, and therefore to show the frequency gap.

4. Conclusions

By applying Bernoulli-Euler equation with the time dependent axial force to the antenna structure resonator, the time term of its vibrational displacement of the antenna structure resonator was found to become the Mathieu equation under a reasonable approximation. We could obtain its frequency gap from the numerical calculation of the stable and unstable regions of the Mathieu equation. From the calculated results, the frequency gap, observed in the antenna structure resonator, was found to be realized by the instable vibration of the elements on which the perturbed time dependent outward axial force exerted from the viewpoint of the Mathieu equation.

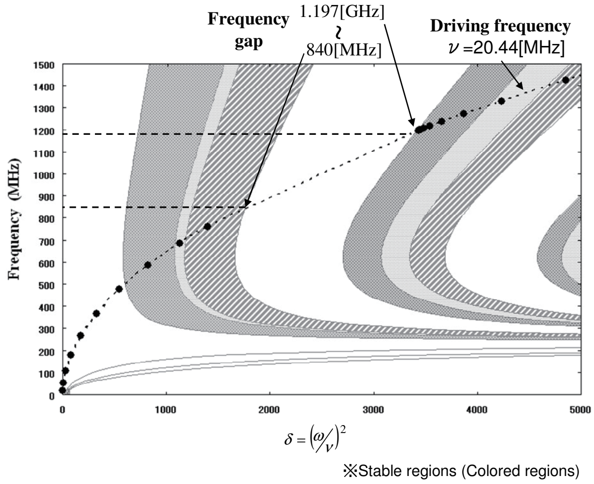

Fig. 2 Relationship between stable and unstable regions of the vibration

(Magnitude of axial force: 5.58×10^{-5} N)

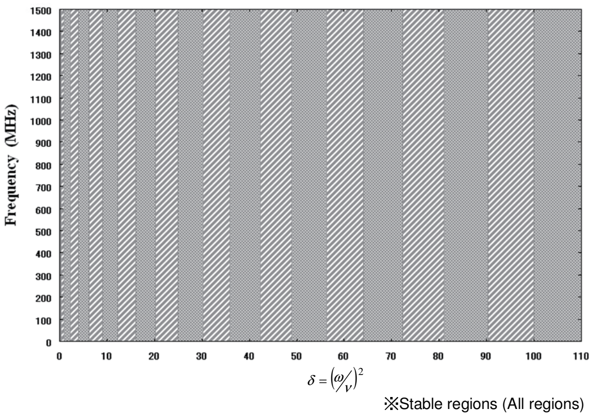

Fig. 3 Relationship between stable and unstable regions of the vibration

(Magnitude of axial force: 5.58×10^{-10} N)

References

[1] H. Itoh, et al.: Jpn. J. Appl. Phys. **47** (2008) 5734.

[2] W. Sochacki: J. Sound Vib. **314** (2008) 180.

[3] A. Gaidarzhy, et al.: Phys. Rev. Lett. **94** (2005) 030402.

[4] D. E. Newland: *Mechanical Vibration Analysis and Computation*, Dover, 2006, pp.420-429.

[5] H. Itoh and H. Tatebe: Jpn. J. Appl. Phys. **49** (2010) 07HB09.