Influence of the Sediment Properties on the Range-Frequency Interference in Shallow Water
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1. Introduction
In the ocean, the propagation of acoustic wave can be described in terms of the acoustic modes supported by the waveguide and each mode will propagate with a slightly different group and phase velocities. This leads to an interference phenomenon producing the striation patterns in spectrogram. Fig. 1 show a typical range-frequency interference in shallow water.

![Fig. 1. Typical range-frequency interference in shallow water.](Image)

The slope of the striations is due to the modal interference and is described by a waveguide invariant parameter \( \beta \) which is a simple function of range, frequency and the slope of the striation.

\[
\beta = \frac{r}{g} \frac{d\omega}{dr} = -\frac{d(1/c_{ph})}{d(1/c_g)} \tag{1}
\]

Where \( r \) is range, \( \omega \) is wave number and \( d\omega/dr \) is the slope of the striation in the spectrogram. As follows from eq. (1), The \( \beta \) is related only to the average value of the derivative \( d(1/c_{ph})/d(1/c_g) \) for a given group of modes. Here, \( 1/c_{ph} \) is modal phase velocity and \( 1/c_g \) is group velocity. The parameter \( \beta \) represents waveguide characteristics and is useful for describing acoustic interference pattern in a waveguide. In recent years, the interference phenomenon of underwater broadband acoustic wave has attracted much interest for applications in estimation of ocean waveguide properties. In this article, the variability of the \( \beta \)-distribution on 4 typical sediment types (clay, silt, sand and gravel) are examined and characterized by moments of the distribution such as skewness and kurtosis.

2. Characterizing the Interference Pattern
A computational method for estimating this slope of striation is the Radon transform which performs line integrals across an interference pattern along different rotation angles and different offsets from the center.

\[
\mathfrak{R}(x, \theta, I(r, \omega)) = \int_{-\infty}^{\infty} I(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta) dy
\]

Where

\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  r \\
  \omega
\end{bmatrix} \tag{2}
\]

The line integral is performed along a line perpendicular to rotated axis and crosses the axis a distance \( x \) away from the reference. Therefore, a line lying at an angle \( \theta \) with respect to the \( \omega \) axis and passing a minimum distance of \( x \) units away from the reference point will be transformed into a point at the coordinates \( (x, \theta) \). Summing along the vertical axis provides the energy per unit angle in the interference pattern along the \( x \) axis for a given \( \theta \) value. To convert the horizontal axis into the unit of \( \beta \), eq. (1) is used. The \( \beta \)-distribution could be characterized by moments of the distributions such as skewness and kurtosis. The skewness characterizes the central tendency of a distribution and the kurtosis is measure of the relative flatness of a distribution.

\[
Kurt(x_1, \ldots, x_N) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{x_i - \bar{x}}{\sigma} \right)^4 \tag{3}
\]

\[
Skew(x_1, \ldots, x_N) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{x_i - \bar{x}}{\sigma} \right)^3
\]

Here, \( \bar{x} \) is the average of distribution and \( \sigma(x_1, \ldots, x_N) \) is standard deviation. \( N \) is the number of integration which is same value of calculated number of \( \theta \). The variability of the \( \beta \)-distribution on 4 typical sediment types is examined. Normal mode code (KRAKEN) was used to compute the acoustic pressure fields using bandwidth of 50Hz at the center frequency of 350 Hz. The geoaoustic parameters of 4 typical sediment types are listed in Table 1. They are taken from compilations of Hamilton geoaoustic model and provide good expectations of what the average properties should be.
Table I. Typical geoacoustic parameters for 4 sediment types.

<table>
<thead>
<tr>
<th>Sediment Type</th>
<th>Clay</th>
<th>Silt</th>
<th>Sand</th>
<th>Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound speed (m/s)</td>
<td>1,500</td>
<td>1,575</td>
<td>1,650</td>
<td>1,800</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>1.5</td>
<td>1.7</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>Attenuation (dB/λ)</td>
<td>0.2</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

3. Results and Discussion

Fig 2. shows the β-distributions of the simulated range-frequency interference patterns and Table II lists estimated skewness and kurtosis of β-distributions for 4 sediment types.

![Normalized Energy vs β for Clay, Silt, Sand, Gravel](image)

Table II. Skewness and kurtosis of β-distributions for 4 sediment types.

<table>
<thead>
<tr>
<th>Sediment Type</th>
<th>Clay</th>
<th>Silt</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness</td>
<td>-1.18</td>
<td>-1.87</td>
<td>-2.28</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.92</td>
<td>5.50</td>
<td>7.12</td>
</tr>
</tbody>
</table>

The location of the peaks and the relative flatness both vary with sediment types. The peak is generally sharper at the hard sediment and the location of the peak, however, has shifted to a slightly lower value of β. Acoustic propagation in the waveguide can be explained by a modal analysis. At hard sediment, more number of modes contributes in the β-distribution than soft sediment. The numbers of mode in 4 different sediment types are shown in Table III.

Table III. Number of propagation modes for the 4 sediment types at the frequency of alentine Hz.

<table>
<thead>
<tr>
<th>Sediment Type</th>
<th>Clay</th>
<th>Silt</th>
<th>Sand</th>
<th>Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of modes</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>13</td>
</tr>
</tbody>
</table>

The higher-order modes provide the detailed, high spatial frequency features in the interference pattern. The higher order modes interact more with the waveguide boundaries so are more strongly attenuated in soft sediment. By diminishing the higher-order modes, the detailed structure in interference pattern is lost, so flattening the peak in the β-distribution (low value of kurtosis). Since the remaining lower-order modes have their upper turning depths in the water column, these modes cause the broadening of the β-distribution, and the amplitude of the distribution at the peak is reduced. The increased value for β is because propagation is dominated by the lower-order acoustic modes. The skewness and kurtosis in gravel sediment were estimated by -2.44, 8.83 and -1.18, 2.92 in clay sediment respectively. In this article, the range-frequency interference pattern is examined by skewness and kurtosis of β-distribution. And this result could be used to estimate the geoacoustic parameters which are composed of sediment. Future work involves the extension of the method developed here to include of geoacoustic parameter inversion.

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References