Non-contact type viscometry using the displacement and phase of liquid surface excited by aerial sound.

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1. Preface
Presently, there are viscometers of several types to measure the viscosity of the liquid, but it takes a lot of time to measure viscosity and handling is troublesome in each viscometer. Those are generally unsuitable for inspection of production line. Therefore, to solve the problems, we have developed a measurement method that derives viscosity from the displacement amplitude on liquid surface. The displacements were measured by non-contact using aerial sound and laser displacement meter. In order to analyze this phenomenon, Voigt model was applied for the vibration of liquid surface. Although it was a great advantage that we could measure viscosity with non-contact, this method was still lack in the convenience, because surface tension and density of liquid had to measure using other measuring instruments. Therefore, in this study, we also measure the phase in addition to the displacement of liquid surface at the same time, and we will find a method to measure directly kinematic viscosity using both measurement values, without using the value of surface tension and density. In addition, we will find a new measurement method to measure easily surface tension and density using this technique at the same time.

2. Experimental method
Figure 1 shows schematic diagram for a viscometer of non-contact type using a low-frequency aerial sound. We used a small loudspeaker with 8cm in dia. as a sound source. The sinusoidal voltage of 100mV with frequency around 10Hz generated by an oscillator was applied to the loudspeaker. The generated sound is then guided through a pipe to the neighboring area on the specimen liquid. The tip of pipe and liquid surface is set at distance of 10mm. The sound is radiated from the tip of pipe with 5mm in dia. at right angle to the surface, and then we measure the displacement by laser displacement meter. Specimen liquid is put into a cylindrical vessel with 6cm in dia. and 2cm in depth, which has absorber inside to prevent standing wave. As a specimen liquid, we used silicone oil with 0.893cSt. 

3. Vibration analysis of liquid surface using vibration model
The principle of this method of viscosity measurement comes from an idea that the displacement and phase of the vibration of liquid surface excited by sound wave (sound pressure) are changed by physical property values such as the liquid’s viscosity, density and surface tension. For the analysis, we try to introduce Voigt model as shown in Figure 2(a) by assuming the vibration of liquid surface to be a mechanical vibration of the viscoelastic body. Figure 2(b) shows an electrical equivalent circuit of series resonance to help our understanding. When the external force of sound pressure is \( f \) and displacement of liquid surface is \( u \), the equivalent mass \( m \) corresponds to density of liquid, the equivalent resistance \( r \) corresponds to viscosity, and the equivalent stiffness \( s \) corresponds to surface tension, respectively. Equation of motion for the vibration model is expressed as the following.

\[
\dot{f}(t) = m \ddot{u}(t) + r \dot{u}(t) + su(t) \quad \ldots(1)
\]

Furthermore, vector equation of Eq. (1) is expressed in the following when \( \omega \) is angular frequency.

\[
f = -\omega^2 m \dot{u} + j \omega r u + su \quad \ldots(2)
\]

Relational expression \( \dot{u} = |\dot{u}| \theta \) of displacement \( u \) and phase \( \theta \) is given by following equation.

\[
\dot{u} = \frac{j \omega \dot{u}}{\sqrt{s - \omega^2 m}} \quad \theta = \tan^{-1} \frac{\omega r}{s - \omega^2 m} \quad \ldots(3)
\]

Furthermore, following equations are obtained from Eq. (3).
Equivalent resistance \( r \) can be directly obtained from Eq. (4). In addition, \( m \) and \( s \) can be obtained from following equations.

\[
m = \frac{A - B}{\omega_a - \omega_b}, \quad s = A + \omega_b^2 m = B + \omega_a^2 m \quad \text{(6)}
\]

where, \( \omega_a \) and \( \omega_b \) are two different angular frequencies, \( A \) is measurement value of Eq. (5) at \( \omega_a \), and \( B \) is measurement value at \( \omega_b \).

\[|\text{U}| = \frac{I}{\text{om}} \sin(\theta) \quad \text{(4)}\]

\[s - \omega_a^2 m = \frac{I}{\text{U}} \cos(\theta) \quad \text{(5)}\]

Because the surface tension of water is larger than it. Therefore, kinematic viscosity can be measured correctly because measurement value of water is on the straight line. Figure 3(b) shows results of displacement \( r \) gotten by Eq. (4) for each kinematic viscosity. Furthermore, an enlarged graph in low viscosity part up to 10cSt and an experimental equation are shown in figure. From the results, it is clear that \( r \) are approximately on a straight line as shown in figure and are proportional to kinematic viscosity. Therefore, kinematic viscosity can be measured by corresponding to the graph shown in Fig. 4(a). Furthermore, it is found that kinematic viscosity of water is measured correctly because measurement value of water is on the straight line. Figure 4(b) shows results of equivalent mass \( m \) and stiffness \( s \) calculated by Eq. (6). The values of \( m \) slightly increase with increasing of kinematic viscosity, and become nearly constant in more than 100cSt. Also, \( m \) of water is larger than it of silicone oil. As the tendency of results almost accorded with it of reference values, we estimated that this measurement method was correct approximately. On the other hand, \( s \) is slightly increases up to 100 cSt and the tendency of \( s \) is almost similar to it of reference. It is found that \( s \) of water is very large in comparison with silicone oil because the surface tension of water is larger than it.

4. Experimental result

Sound waves radiated on liquid surface need two different frequencies to calculate parameter \( m \) and \( s \) from Eq. (6). Therefore, we used two of 10Hz and 12Hz. Figure 3(a) shows results of displacement of liquid surface for kinematic viscosity. Figure 3(b) shows phase difference of between the displacement of liquid surface and sound wave. The measurement results of water are also shown in those graphs. From these results, it is found that the displacements decrease with increasing of kinematic viscosity of silicone oil and phase differences increase. Figure 4(a) shows result of \( r \) gotten by Eq. (4) for each kinematic viscosity. Furthermore, an enlarged graph in low viscosity part up to 10cSt and an experimental equation are shown in figure. From the results, it is clear that \( r \) are approximately on a straight line as shown in figure and are proportional to kinematic viscosity. Therefore, kinematic viscosity can be measured by corresponding to the graph shown in Fig. 4(a). Furthermore, it is found that kinematic viscosity of water is measured correctly because measurement value of water is on the straight line. Figure 4(b) shows results of equivalent mass \( m \) and stiffness \( s \) calculated by Eq. (6). The values of \( m \) slightly increase with increasing of kinematic viscosity, and become nearly constant in more than 100cSt. Also, \( m \) of water is larger than it of silicone oil. As the tendency of results almost accorded with it of reference values, we estimated that this measurement method was correct approximately. On the other hand, \( s \) is slightly increases up to 100 cSt and the tendency of \( s \) is almost similar to it of reference. It is found that \( s \) of water is very large in comparison with silicone oil because the surface tension of water is larger than it.

5. Conclusion

In the non-contact type viscometer that has been studied by us, we have been calculated kinematic viscosity using Voigt model for the vibration displacement of specimen liquid surface. But this method was lack in the convenience, because it is necessary to measure the surface tension and density of liquid by another instrument. We tried to examine therefore a measurement method of viscosity using both of the displacement and newly-introduced the phase of liquid surface excited by aerial sound. As a result, it was clarified that the kinematic viscosity could be obtained directly from the displacement and phase without referring to density and surface tension. It was a great result that we developed the excellent viscometer in convenience. Furthermore, it was revealed that density and surface tension of specimen liquid could be obtained concurrently with kinematic viscosity. In the near future, we will investigate whether ultrasonic sound source is useable for the purpose of enlarging distance between sound source and specimen liquid surface.

References
3. Catalog of Silicone fluid: Shin-Etsu Chemical Co. Ltd.