Elastic Constants of Rare Earth Bismuth Iron Garnet (111) Films Measured by Resonant-Ultrasound Spectroscopy

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1. Introduction

Rare earth bismuth iron garnet (referred to as BIG hereinafter) is a candidate of the Faraday rotator, because its Faraday rotation coefficient is larger than the conventional materials such as yttrium iron garnet. It enables us to decrease the thickness of the rotator (< 500 μm), and downsizing the optical devices. The optical devices such as the isolator are often affected by the thermal stress due to the temperature change during working, which disarranges the optical path and their performance. In order to demonstrate the best performance, we have to simulate the thermal stress accurately. Elastic constants $C_{ij}$ of the optical crystals are then indispensable, and reliable value of $C_{ij}$ is desired. $C_{ij}$ of garnet and related crystals have been reported. However, the complete set of $C_{ij}$ of a BIG crystal has never been reported. Also, measurement of the thin film $C_{ij}$ is not straightforward. Furthermore, the BIG thin films are usually grown so that its $(<111)$ direction is parallel to the thickness direction, which makes the measurement of the $C_{ij}$ difficult described below. In this study, we solve this problem using the resonant-ultrasound spectroscopy (RUS) and determine the complete set of $C_{ij}$ of BIG single crystal for the first time.

2. Specimens

$(\text{GdHoBi})_3(\text{FeGaAl})_5\text{O}_{12}$ was grown on $(\text{GdCa})_3(\text{GaMgZr})_5\text{O}_{12}$ substrates by the liquid-phase-epitaxy (LPE) method. The BIG single crystal shows cubic symmetry (space group $Ia\overline{3}d$). The $(<111>)$ direction is aligned in the film-thickness direction. Cubic-symmetric crystal possesses three independent $C_{ij}$ as shown in later in Eq.(1). After removing the substrate, BIG films were cut into rectangular-parallelepiped specimens. Their edges are parallel to the $[111]$, $[110]$, and $[211]$ crystallographic axes. We changed the content of Pb and prepared two kinds of BIG; one contains 0.1-mass % Pb, and the other 0.5-mass % Pb. Four specimens were prepared for each material.

3. Resonant-ultrasound spectroscopy

Resonance frequencies of mechanical free vibration of a rectangular parallelepiped depend on the mass density, dimensions, and $C_{ij}$. Therefore, $C_{ij}$ are inversely determined by measuring the resonance frequencies with other parameters. Usual RUS method requires a specimen whose edges are parallel to the principal crystallographic axes that gives the highest vibrational symmetry. However, BIG crystals grow epitaxially with $(<111>)$ direction parallel to the film-thickness directions, and fabrication of $[001]$-oriented rectangular parallelepiped specimens is unrealistic.

When $x_1$, $x_2$, and $x_3$ axes are defined to be parallel to the $<100>$, $<010>$, and $<001>$ axes, respectively, stress-strain relationship of BIG crystal is expressed as

$$
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{pmatrix} =
\begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{13} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{pmatrix}
$$

(1)

Where, $\sigma$ and $\varepsilon$ denote the stress and engineering strain, respectively. When the orthogonal coordinate axes are in the $(<110>)$, $(<211>)$, and $(<111>)$
crystallographic directions, the stress-strain relationship becomes,

\[
\begin{bmatrix}
\sigma_1' \\
\sigma_2' \\
\sigma_3' \\
\sigma_4' \\
\sigma_5' \\
\sigma_6'
\end{bmatrix} =
\begin{bmatrix}
C_{11}' & C_{12}' & C_{13}' & 0 & 0 & 0 \\
C_{12}' & C_{22}' & C_{23}' & 0 & 0 & 0 \\
C_{13}' & C_{23}' & C_{33}' & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44}' & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44}' & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}'
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1' \\
\varepsilon_2' \\
\varepsilon_3' \\
\varepsilon_4' \\
\varepsilon_5' \\
\varepsilon_6'
\end{bmatrix}
\]  

(2)

This apparently corresponds to the stress-strain relationship of a trigonal-symmetric crystal; the six $C_{ij}'$ are expressed by the combination of the three independent $C_{ij}$ in Eq. (1).

Considering that RUS has been applied to the trigonal single crystals such as quartz, we can determine the cubic-symmetric $C_{ij}$ of (111) films using the RUS algorithm for the trigonal materials. In the analysis, we have to be careful about the number of vibrational group. When the edges of the specimen are parallel to the <100> crystallographic axes, the resonance frequencies are divided into eight vibrational groups. However, a trigonal-symmetric crystal shows lower elastic symmetry, and the number of the vibrational groups decreases to four.

Here, we describe the measurement procedure of the RUS. Resonance frequencies are calculated assuming the three $C_{ij}$ by Rayleigh-Ritz method, and measured resonance frequencies are homologized with them. We then inversely determine the best-fitting $C_{ij}$ using the least-square method. Resonance frequencies are measured using the tripod transducer, and vibrational mode of each resonance frequency is identified using the laser-Doppler interferometry.

4. Results

Figure 1 shows a typical resonance spectrum. We measure more than 37 resonance frequencies, and three $C_{ij}$ are determined from them. Figure 2 shows the $C_{ij}$ of BIG crystals. Fluctuations of $C_{ij}$ among the four specimens are smaller than 1.9%, which indicates that reliable $C_{ij}$ are obtained by the RUS. $C_{ij}$ of 0.1%-Pb BIG crystal are well in agreement with those of 0.5%-Pb BIG, and significant Pb-content dependence of $C_{ij}$ was not observed. Furthermore, they are approximately close to the $C_{ij}$ of yttrium iron garnet (YIG) and terbium iron garnet (TIG).

Elastic anisotropy factor of BIG crystals, $A=2C_{44}/(C_{11}+C_{12})$, is 0.99, which indicates that this crystal is nearly isotropic. The anisotropy factor of YIB and TIB is reported to be also close to isotropic, $A_{\text{YIG}}=0.95$ and $A_{\text{TIG}}=1.00$, and we consider that isotropic elasticity is a common characteristic in a series of garnet crystals.

References